KEDAH (2006)
MATHEMATICS T (PAPER 1)

1. Given that the line $y = mx$ cuts the curve $x^2 + y^2 + 4x + 2y - 20 = 0$ at the points P and Q. If M is the midpoint of the chord PQ, find the coordinates of M, in terms of m. Hence, find the equation of the locus of M, as m varies. [5]

2. Evaluate $\sum_{r=1}^{n} \frac{(-1)^{r-1}x^{r-1}}{3^{r-2}}$.

   Given that the series is convergent, state the range of values of x, and find the sum to infinity in terms of x. [6]

3. Given that $\mathbf{A} = \begin{pmatrix} 1 & 7 & 3 \\ 0 & 5 & 2 \\ 3 & 0 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & -7 & 1 \\ -6 & 7 & 2 \\ 15 & -21 & -8 \end{pmatrix}$. Show that $\mathbf{AB} + 3\mathbf{A} - 8\mathbf{I} = \mathbf{0}$, where $\mathbf{I}$ is the 3 x 3 identity matrix and $\mathbf{0}$ is the 3 x 3 null matrix. Hence, deduce $\mathbf{A}^{-1}$. [6]

4. Show that the equation of the tangent to the curve $2x^2e^{2y} - e^{-y} = 0$ at the point where $x = 2$ is $3y + x = 2 - \ln 8$. [6]

5. The function f is defined as $f(x) = \begin{cases} k\sin x & , x \le \frac{1}{2}\pi \\ (x-\pi)^2 + \frac{1}{2}\pi^2 & , x > \frac{1}{2}\pi \end{cases}$.

   Find the value of k if f is continuous at $x = \frac{\pi}{2}$. [2]

   With the value of k obtained, sketch the graph of f and determine if f is a one to one function. [5]

6. Functions g and h are defined by $g : x \to \ln x, x > 0$ and $h : x \to \sqrt{x-2}, x > 2$.

   (a) Express the composite function gh^{-1} in similar form. [3]

   (b) By using the trapezium rule with 5 ordinates, find an approximation for $\int_0^1 k(x)\,dx$, where $k(x) = gh^{-1}(x)$, correct to 2 decimal places. [4]

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7. Show that $\int_2^3 \frac{x}{(x^2-1)^2}\,dx = \frac{5}{48}$. [2]

   Using integration by parts, show that $\int \frac{x}{(x^2-1)^2}\,dx = \frac{x^2}{2(x^2-1)^2} + \int \frac{2x^3}{(x^2-1)^3}\,dx$. [2]

   Hence, find the value of $\int_2^3 \frac{x^3}{(x^2-1)^3}\,dx$. [4]

8. Sketch on the same diagram, the graphs of $y = \frac{1}{x}$, and $3y = 2|x| - 5$.

   Hence, find the range of values of x such that $2|x| - 5 \ge \frac{3}{x}$. [8]

9. Given that $p(x) = 5 - 3x - 2x^2, x \in \mathcal{R}$.

   (a) Express $p(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants whose values are to be determined. Hence, find the maximum value of $p(x)$. [4]

   (b) If $p(x)$ is a factor of the polynomial $q(x) = -2x^4 - 5x^3 + kx^2 + 2x + 5$, determine the value of k.
   With this value of k, find all the zeroes of $q(x)$. [6]

10. Sketch the graph of $y = 1 + \frac{4}{x}$. By sketching a suitable graph on the same coordinates axes, show that the equation $xe^x - 4 = 0$ has exactly one real root, α. [5]

    Using the Newton-Rapson method with $\alpha_0 = 1.5$ as the first approximation, find the value of α correct to three decimal places. [6]

11. Find the points of intersection between the straight line $2y = 6 - x$ and the curve $2y = 4x^3 - 20x^2 + 23x + 6$. [5]

    Sketch, on a same diagram, the straight line $2y = 6 - x$ and the curve $2y = 4x^3 - 20x^2 + 23x + 6$.
    Hence, find the area of the region enclosed by the straight line and the curve. [7]

12. Expand $(1-x)^{-p}(1-x^2)^{-q}$ in ascending powers of x up to and including the terms in x^3. [6]

    (a) If the first three terms in the above expansion are $1 - 2x + \frac{10}{9}x^2$, find the values of p and q. [3]

    (b) If $q = -\frac{1}{8}$ and the value of p is such that the coefficient of x^2 has the smallest value, show that the coefficient of x^3 in the above expansion is equal to zero. [5]

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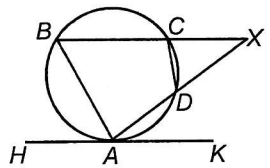
MATHEMATICS T (PAPER 2)

1. Find the acute angle between any pair of faces of a regular tetrahedron. [5]

2. Given that $\frac{dy}{dx} = \frac{xy}{1-x}$ and $y = 2$ when $x = 0$. Find y in terms of x. [6]

3. Solve the equation $\sin 2x - 3\cos^2 x + 1 = 0$, for $0^\circ < x < 360^\circ$. [7]

4. In the diagram, HAK is a tangent to the circle $ABCD$. The chord BC is parallel to HK. Chords AD and BC produced meet at X. Prove that



(i) $\angle ABC = \angle ADB$, [3]

(ii) ΔABD and ΔCXD are similar. [4]

5. A steel bar is heated to a temperature of 700°C and dropped into a drum of powdered ice. The temperature falls to 500°C in 30 seconds. The rate of cooling is proportional to the temperature of the steel bar.
Express this information as a differential equation connecting T and t, where T is the temperature of the bar at time t. [1]

Solve this equation and express T in terms of t. [6]

Find the temperature of the bar after an additional 3 minutes. [3]

6. The position vectors of points A, B with respect to the origin, O are $\underline{a}$ and $\underline{b}$ respectively. Show that for all real values of k, the point P with position vector

$\underline{p} = k\underline{a} + (1-k)\underline{b}$ is collinear with A and B. [3]

(i) Deduce that the point Q with position vector $\underline{q} = 3\underline{a} - 2\underline{b}$ lies on BA produced, and find the ratio $QA : AB$. [4]

(ii) The point R has position vector $\underline{r} = 3\underline{a} + 5\underline{b}$. Find the position vector of the point where OR meets AB (produced if necessary). [5]

(iii) Prove that the value of k for which OP and AB are perpendicular is given by $k = \frac{\underline{b}\bullet(\underline{b}-\underline{a})}{(\underline{b}-\underline{a})\bullet(\underline{b}-\underline{a})}$. [3]

7. An athlete owns six pairs of shoes. He chooses four shoes at random.

(i) Find the probability that 4 left shoes are selected. [2]

(ii) Find the probability that 2 left shoes and 2 right shoes are selected. [2]

4

8. The probability that Mimi dials a wrong number when making a telephone call is 0.015. In a week, Mimi makes 50 telephone calls. Assuming that the events are independent and using a suitable approximation, find the probability that in such a week

(i) Mimi dials no wrong numbers, [3]

(ii) Mimi dials more than two wrong numbers. [2]

Comment on the assumption made. [1]

9. The table shows the number of persons according to their ages in years, who watched a film in a mini theatre.

Age (x)	Frequency
$15 \le x < 18$	14
$18 \le x < 21$	38
$21 \le x < 27$	44
$27 \le x < 33$	16
$33 \le x < 39$	6
$39 \le x < 45$	2

(i) Plot a histogram to illustrate the data. [3]

(ii) Calculate the median and the semi-interquartile range. [4]

10. A discrete random variable X has the probability distribution given in the table.

x	2	3	4	5
$P(X = x)$	p	$\frac{2}{10}$	$\frac{3}{10}$	q

Given that $E(X) = 4$, find the values of p and q. [4]

Show that $Var(X) = 1$, and evaluate $E(|X-4|)$. [4]

11. Pepsi Cola is sold in bottles of two sizes, standard and large. For each size, the content, in litres, of a randomly chosen bottle is normally distributed with means and standard deviation (σ) shown in the table.

	Mean	σ
Standard	0.760	0.008
Large	1.010	0.009

(i) Show that the probability that a randomly chosen standard bottle contains less than 0.750 litres is 0.1056. [2]

(ii) Find the probability that a box of 10 randomly standard bottles contains at least three bottles whose contents are each less than 0.750 litres. [4]

(iii) Find the probability that there is more cola in four randomly chosen standard bottles than in three randomly chosen large bottles. [5]

12. The continuous random variables X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{(x+1)^4} & , x \ge 0 \\ 0 & , x < 0 \end{cases}$$, where k is a constant.

(i) Show that $k = 3$, and find the cumulative distribution function of X. Find also the value the x such that $P(X \le x) = \frac{7}{8}$. [6]

(ii) Evaluate $E(X+1)$, and deduce that $E(X) = \frac{1}{2}$. [4]

(iii) By considering $Var(X+1)$, find the value of $Var(X)$. [4]

KEDAH 06

No:..... MT1 Date:.....

1. For P, Q: $x^2+(mx)^2+4x+2(mx)-20=0$

$\therefore (1+m^2)x^2+(4+2m)x-20=0$

$\therefore x_1+x_2=-\frac{B}{A}=-\frac{2(2+m)}{1+m^2}$

$\therefore M=\left(\frac{x_1+x_2}{2}, mx_M\right)=\left(-\frac{2+m}{1+m^2}, -\frac{m^2+2m}{1+m^2}\right)$ ✱

$\therefore x\left(1+\left(\frac{y}{x}\right)^2\right)=-\left(2+\frac{y}{x}\right)$

$\therefore x^2+y^2+2x+y=0$ ✱ [locus of M]

2. $\sum_{r=1}^{n}\frac{(-1)^{r-1}x^{r-1}}{3^{r-2}}=3\sum_{r=1}^{n}\left(-\frac{1}{3}x\right)^{r-1}=3\left\{\frac{(1)[1-(-\frac{1}{3}x)^n]}{1+\frac{1}{3}x}\right\}$

$=\frac{9}{3+x}\left[1-\left(-\frac{1}{3}x\right)^n\right]$ ✱

For S_n converges: $|r|<1 \Rightarrow \left|-\frac{1}{3}x\right|<1$

$\therefore |x|<3 \Rightarrow \{x: -3<x<3\}$ ✱

$S_\infty=3\left[\frac{1}{1-(-\frac{1}{3}x)}\right]=\frac{9}{3+x}$ ✱

3. $AB+3A-8I=\begin{bmatrix}1&7&3\\0&5&2\\3&0&-1\end{bmatrix}\begin{bmatrix}2&-7&1\\-6&7&2\\15&-21&-8\end{bmatrix}+3A-8I$

$=\begin{bmatrix}5&-28&-9\\0&-7&-6\\-9&0&11\end{bmatrix}+3\begin{bmatrix}1&7&3\\0&5&2\\3&0&-1\end{bmatrix}-8\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=0$

$\therefore AB+3A-8I=0$ ✱ —$(\times A^{-1})$

$IB+3I-8A^{-1}=0 \Rightarrow A^{-1}=\frac{1}{8}[B+3I]$

$\therefore A^{-1}=\frac{1}{8}\begin{bmatrix}5&-7&1\\-6&10&2\\15&-21&-5\end{bmatrix}$ ✱

4. $2x^2e^{2y}-e^{-y}=0 \Rightarrow 2x^2=e^{-3y}$

$4x=-3e^{-3y}\left(\frac{dy}{dx}\right)$ $(x=2, y=-\ln 2)$

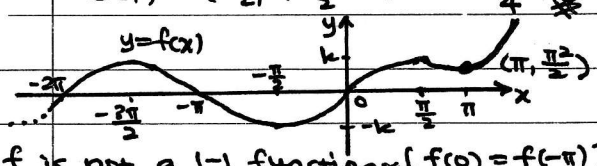
$x=2$: $8=-3(8)\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=-\frac{1}{3}$

$\therefore$ tangent: $\frac{y+\ln 2}{x-2}=-\frac{1}{3}$

$\therefore 3y+x=3-\ln 8$ ✱, $[3\ln 2=\ln 8]$

5. f continuous at $x=\frac{\pi}{2}$: $f(\frac{\pi}{2})=\lim_{x\to\frac{\pi}{2}^+}f(x)$

$k(1)=(-\frac{\pi}{2})^2+\frac{\pi^2}{2} \Rightarrow k=\frac{3\pi^2}{4}$ ✱



f is not a 1-1 function ✱ $[f(0)=f(-\pi)]$

6. (a) $h^{-1}: x\to x^2+2,\ x>0$

$\therefore gh^{-1}(x)=g(x^2+2)=\ln(x^2+2)$

$\therefore gh^{-1}: x\to \ln(x^2+2),\ x>0$ ✱

(b) $\int_0^1 k(x)\,dx=\frac{1}{2}(0.25)[\ln 2+\ln 3+2(\ln 2.0625+\ln 2.25+\ln 2.5625)]$

$=0.53$ ✱ (2 dp)

7. $\int_2^3\frac{x}{(x^2-1)^2}dx=\frac{1}{2}\int_2^3\frac{2x}{(x^2-1)^2}dx=\frac{1}{2}\left[\frac{-1}{x^2-1}\right]_2^3$

$=-\frac{1}{2}\left[\frac{1}{8}-\frac{1}{3}\right]=\frac{5}{48}$ ✱

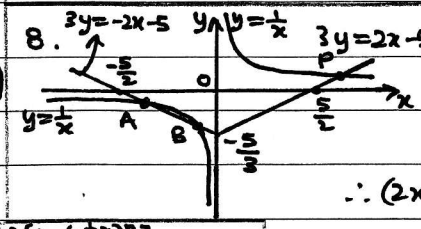
$\int\frac{x}{(x^2-1)^2}dx=\frac{x^2}{2}\left[\frac{1}{(x^2-1)^2}\right]-\int\frac{x^2}{2}\left[\frac{-2(2x)}{(x^2-1)^3}\right]dx$

$=\frac{x^2}{2(x^2-1)^2}+\int\frac{2x^3}{(x^2-1)^3}dx$ ✱

$\therefore \int_2^3\frac{x^3}{(x^2-1)^3}dx=\frac{1}{2}\left[\int_2^3\frac{x}{(x^2-1)^2}dx-\left[\frac{x^2}{2(x^2-1)^2}\right]_2^3\right]$

$=\frac{1}{2}\left[\frac{5}{48}-\left(\frac{9}{2(8)^2}-\frac{4}{2(3)^2}\right)\right]=\frac{295}{2304}$ ✱

8.



For A, B:

$3(\frac{1}{x})=-2x-5$

$\therefore 2x^2+5x+3=0$

$\therefore (2x+3)(x+1)=0$

$\therefore x=-\frac{3}{2}$ or -1

For P: $3(\frac{1}{x})=2x-5$

$\therefore 2x^2-5x-3=0 \Rightarrow (2x+1)(x-3)=0$

$[\because x_P>0]$: $x_P=3$

For $2|x|-5\geq\frac{3}{x}$: $\frac{2|x|-5}{3}\geq\frac{1}{x}$

$\therefore \{x: x\leq-\frac{3}{2}$ or $-1\leq x<0$ or $3\leq x\}$ ✱

9. (a) $p(x)=-2(x^2+\frac{3}{2}x)+5=-2[(x+\frac{3}{4})^2-\frac{9}{16}]+5$

$=-2(x+\frac{3}{4})^2+6\frac{1}{8}$ ✱

$\therefore p(x)_{max}=p(-\frac{3}{4})=6\frac{1}{8}$ ✱

(b) $q(x)=-2x^4-5x^3+10x^2+2x+5$

$\equiv(-2x^2-3x+5)(x^2+x+1)$

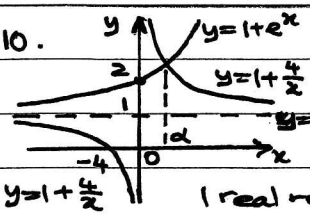
$[x^2]$: $k=-2-3+5=0$ ✱

$\therefore$ For $q(x)=0$: $(5+2x)(1-x)(x^2+x+1)=0$

$\therefore x=-\frac{5}{2}, 1$ or $\frac{1}{2}[-1\pm\sqrt{3}i]$

$\therefore$ zeroes of $q(x)$ are $-\frac{5}{2}, 1$ and $\frac{1}{2}[1\pm\sqrt{3}i]$ ✱

10.



① $xe^x-4=0 \Rightarrow e^x=\frac{4}{x}$

$\therefore e^x+1=1+\frac{4}{x}$

$\therefore$ equation ① has only 1 real root (α), since exists only 1 intersection point. ✱ $\left[\begin{array}{l}xe^x-4=0\\ \therefore x-4e^{-x}=0\end{array}\right]$

Let $f(x)=x-4e^{-x}$

$f'(x)=1+4e^{-x}$

By NRM: $\alpha_0=1.5$

$\alpha_1=1.5-\frac{f(1.5)}{f'(1.5)}=1.1790\simeq1.179$

$\alpha_2=1.1790-\frac{f(1.1790)}{f'(1.1790)}=1.2020\simeq1.202$

$\alpha_3=1.2020-\frac{f(1.2020)}{f'(1.2020)}=1.2022\simeq1.202$

$\therefore \alpha=1.202$ ✱ (3 dp)

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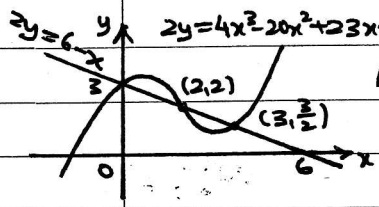
11. $2y = 6 - x$ — ①, $2y = 4x^3 - 20x^2 + 23x + 6$ — ②

② − ①: $4x^3 - 20x^2 + 24x = 0$

$\therefore\ 4x(x-2)(x-3) = 0 \Rightarrow x = 0, 2$ or 3

$\therefore$ Intersection points $= (0,3), (2,2), (3, \frac{3}{2})$ ✱



Area $= \int_0^2 (2x^3 - 10x^2 + 12x)\,dx - \int_2^3 [2x^3 - 10x^2 + 12x]\,dx$

$= \left[\frac{1}{2}x^4 - \frac{10}{3}x^3 + 6x^2\right]_0^2 - \left[\frac{1}{2}x^4 - \frac{10}{3}x^3 + 6x^2\right]_2^3$

$= 2\left[8 - \frac{80}{3} + 24\right] - 0 - \left[\frac{81}{2} - 90 + 54\right] = \frac{37}{6}$ ✱

12. $(1-x)^{-p}(1-x^2)^{-q} = \left[1 - p(-x) + \frac{(-p)(-p-1)}{2}(-x)^2 + \frac{(-p)(-p-1)(-p-2)}{6}(-x)^3 + \dots\right]\left[1 - q(-x^2) + \dots\right]$

$= \left[1 + px + \frac{p(p+1)}{2}x^2 + \frac{p(p+1)(p+2)}{6}x^3 + \dots\right]$

$\left[1 + qx^2 + \dots\right] = 1 + px + \left[q + \frac{p(p+1)}{2}\right]x^2$

$+ \left[pq + \frac{1}{6}p(p+1)(p+2)\right]x^3 + \dots$ ✱

(a) $[x]$: $p = -2$ ✱

$[x^2]$: $q + \frac{(-2)(-1)}{2} = \frac{10}{9} \Rightarrow q = \frac{1}{9}$ ✱

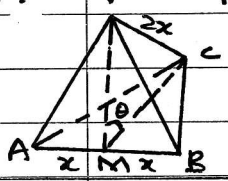
(b) $q = -\frac{1}{8}$: $f(p) = \frac{1}{2}\left[p^2 + p - \frac{1}{4}\right]$

$\therefore f(p)_{min}$: $f'(p) = 0 \Rightarrow 2p + 1 = 0$

$\therefore p = -\frac{1}{2}$

coef. of $x^3 = -\frac{1}{8}(-\frac{1}{2}) + \frac{1}{6}(-\frac{1}{2})(\frac{1}{2})(\frac{3}{2}) = 0$ ✱

1. $M = M_{AB}$ and $VM \perp AB$, $CM \perp AB$



$VM = CM = x\tan 60^\circ = x\sqrt{3}$

$\cos\theta = \frac{3x^2 + 3x^2 - 4x^2}{2(x\sqrt{3})(x\sqrt{3})} = \frac{1}{3}$

$\therefore \theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5^\circ$ ✱

2. $\frac{dy}{dx} = \frac{xy}{1-x} \Rightarrow \int \frac{1}{y}\,dy = \int \left[\frac{1}{1-x} - 1\right]dx$

$\therefore \ln y = -\ln(1-x) - x + c$ — ①

$x = 0, y = 2$: $\ln 2 = c$ — ②

① − ②: $\ln\left(\frac{y}{2}\right) = -\ln(1-x) - x$

$\therefore\ y = \frac{2}{(1-x)e^x}$ ✱

3. $\sin 2x - 3\cos^2 x + 1 = 0$

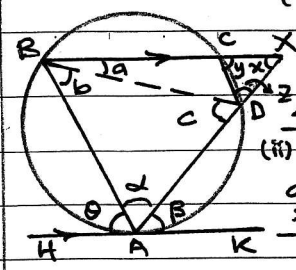
$\therefore\ 2sc - 3c^2 + (s^2 + c^2) = 0$

$\therefore\ \tan^2 x + 2\tan x - 2 = 0$

$\therefore\ \tan x = \frac{-2 \pm \sqrt{4+8}}{2} = \tan 36.2^\circ, -\tan 69.9^\circ$

$\therefore x = 36.2^\circ, 110.1^\circ, 216.2^\circ, 290.1^\circ$ ✱

4.



Statement	Reason
(i) $\theta = c$	∠s in alternate segment.
$\theta = a + b$ $\therefore c = a + b$ $\angle ABC = \angle ADB$	Alternate ∠s (HK // BC) ✱
(ii) $\alpha = y$	Ext. ∠ of cyclic ABCD
$a + b = z$ $\therefore c = z$	Ext. ∠ of cyclic ABCD
$b = \beta$	∠s in alternate segment.
$\beta = x$ $\therefore b = x$	Alternate ∠s
$\therefore \triangle ABD \sim \triangle CXD$ ✱	

5. $\frac{dT}{dt} \propto T$: $\frac{dT}{dt} = -kT$, $(k > 0)$ ✱

$\int \frac{1}{T}\,dT = \int -k\,dt \Rightarrow \ln T = -kt + c$ — ①

$t = 0, T = 700$: $\ln 700 = c$ — ②

① − ②: $\ln\left(\frac{T}{700}\right) = -kt$

$t = 30, T = 500$: $\ln\left(\frac{5}{7}\right) = -30k$

$\therefore \ln\left(\frac{T}{700}\right) = -\frac{t}{30}\ln\left(\frac{7}{5}\right)$

$\therefore T = 700\left(\frac{7}{5}\right)^{-t/30}$ ✱

$t = 210$: $T = 700\left(\frac{7}{5}\right)^{-7} = 66.4^\circ C$ ✱

6. $\underline{p} = k\underline{a} + (1-k)\underline{b} \Rightarrow \underline{p} - \underline{b} = k(\underline{a} - \underline{b})$

$\therefore \overrightarrow{OP} - \overrightarrow{OB} = k[\overrightarrow{OA} - \overrightarrow{OB}] \Rightarrow \overrightarrow{BP} = k\overrightarrow{BA}$

$\therefore$ BP // BA with a common point B.

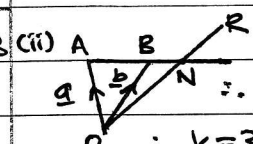
$\therefore$ B, P, A are collinear ✱

(i) $\overrightarrow{QA} = \underline{a} - (3\underline{a} - 2\underline{b}) = 2(\underline{b} - \underline{a}) = 2\overrightarrow{AB}$

$\therefore$ QA // AB with a common point A.

$\therefore$ Q lies on BA produced, QA : AB = 2 : 1 ✱

(ii)



let $\overrightarrow{ON} = m\overrightarrow{OR}$

$\therefore k\underline{a} + (1-k)\underline{b} = m(3\underline{a} + 5\underline{b})$

$\therefore k = 3m$ and $1 - k = 5m$

$\therefore 1 - 3m = 5m \Rightarrow m = \frac{1}{8}$

$\therefore \overrightarrow{ON} = \frac{1}{8}[3\underline{a} + 5\underline{b}]$ ✱

(iii) $\overrightarrow{OP} \perp \overrightarrow{AB}$: $\overrightarrow{OP} \cdot \overrightarrow{AB} = 0$

$\therefore [k\underline{a} + (1-k)\underline{b}] \cdot [\underline{b} - \underline{a}] = 0$

$\therefore [\underline{b} - k(\underline{b} - \underline{a})] \cdot [\underline{b} - \underline{a}] = 0$

$\underline{b} \cdot (\underline{b} - \underline{a}) - k(\underline{b} - \underline{a}) \cdot (\underline{b} - \underline{a}) = 0$

$\therefore k(\underline{b} - \underline{a}) \cdot (\underline{b} - \underline{a}) = \underline{b} \cdot (\underline{b} - \underline{a})$

$\therefore k = \frac{\underline{b} \cdot (\underline{b} - \underline{a})}{(\underline{b} - \underline{a}) \cdot (\underline{b} - \underline{a})}$ ✱

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7. (a) $\text{Prob} = \frac{{}^6C_4}{{}^{12}C_4} = \frac{1}{33}$ ✱

(b) $\text{Prob} = \frac{{}^6C_2 \cdot {}^6C_2}{{}^{12}C_4} = \frac{5}{11}$ ✱

8. Let X = no. of wrong dials

$X \sim B(50, 0.015) \rightarrow X \sim P_o(0.75)$

(i) $P(X=0) = e^{-0.75} = 0.4724$ ✱

(ii) $P(X>2) = 1 - e^{-0.75}\left[1 + 0.75 + \frac{0.75^2}{2}\right]$

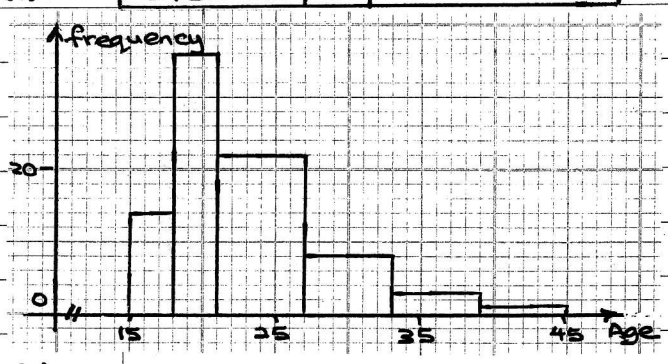
$= 0.0405$ ✱

Assumption not valid, since wrong dialing with the same tel. no. are not independent. ✱

9.

x	f	Height of Histogram
$15 \le x < 18$	14	14
$18 \le x < 21$	38	38
$21 \le x < 27$	44	22
$27 \le x < 33$	16	8
$33 \le x < 39$	6	3
$39 \le x < 45$	2	1

(i)



(ii)

$m = x_{60} = 21 + \left(\frac{60-52}{44}\right)(6) = 22.09$ ✱

$Q_1 = x_{30} = 18 + \left(\frac{30-14}{38}\right)(3) = 19\frac{5}{19}$

$Q_3 = x_{90} = 21 + \left(\frac{90-52}{44}\right)(6) = 26\frac{2}{11}$

$\therefore \text{SIQR} = \frac{1}{2}(Q_3 - Q_1) = 3.46$ ✱

10. $\sum P(X=x) = 1$: $p + q = 0.5$

$E(X) = 4$: $2p + 0.6 + 1.2 + 5q = 4$

$\therefore 2(0.5 - q) + 5q = 2.2$

$\therefore q = 0.4$ ✱ $\Rightarrow p = 0.1$ ✱

$E(X^2) = 0.1(4) + 0.2(9) + 0.3(16) + 0.4(25)$

$= 17$

$\therefore Var(X) = 17 - (4)^2 = 1$ ✱

$E(|X-4|) = 0.1(2) + 0.2(1) + 0 + 0.4(1)$

$= 0.8$ ✱

11. Let S = Standard, L = Large

(i) $P(S < 0.75) = P\left(Z < \frac{0.75 - 0.76}{0.008}\right)$

$= P(-1.25) = 0.1056$ ✱

(ii) Let X = no. of bottles contain < 0.75

$X \sim B(10, 0.1056)$

$P(X \geq 3) = 1 - [P(X = 0, 1, 2)]$

$= 1 - [0.8944^{10} + 10(0.8944)^9(0.1056)$

$+ 45(0.8944)^8(0.1056)^2] = 0.0802$ ✱

(iii) Let $T = (S_1 + S_2 + S_3 + S_4) - (L_1 + L_2 + L_3)$

$\therefore T \sim N(4(0.76) - 3(1.01), 4(0.008)^2 + 3(0.009)^2)$

$\therefore T \sim N(0.01, 0.000499)$

$P(S_1 + S_2 + S_3 + S_4 > L_1 + L_2 + L_3)$

$= P(T > 0) = P\left(Z > \frac{0 - 0.01}{\sqrt{4.99 \times 10^{-4}}}\right)$

$= R(-0.4477) = 0.6729$ ✱

12. (i) $\int_0^\infty \frac{k}{(x+1)^4}\,dx = 1 \Rightarrow \left[\frac{-k}{3(x+1)^3}\right]_0^\infty = 1$

$\therefore 0 + \frac{k}{3} = 1 \Rightarrow k = 3$ ✱

$P(X \le x) = \left[\frac{-1}{(x+1)^3}\right]_0^x = 1 - \frac{1}{(x+1)^3}$

$\therefore F(x) = \begin{cases} 0 & x < 0 \\ 1 - \frac{1}{(x+1)^3} & x \geq 0 \end{cases}$ ✱

For $P(X \le x) = \frac{7}{8}$: $1 - \frac{1}{(x+1)^3} = \frac{7}{8}$

$\therefore (x+1)^3 = 8 \Rightarrow x = 2 - 1 = 1$ ✱

(ii) $E(X+1) = \int_0^\infty \frac{3}{(x+1)^3}\,dx = \frac{3}{2}\left[\frac{-1}{(x+1)^2}\right]_0^\infty$

$= \frac{3}{2}[0 - (-1)] = \frac{3}{2}$ ✱

$E(X+1) = E(X) + 1$: $\frac{3}{2} = E(X) + 1$

$\therefore E(X) = \frac{1}{2}$ ✱

(iii) $E(X+1)^2 = \int_0^\infty \frac{3}{(x+1)^2}\,dx = \left(\frac{-3}{x+1}\right)_0^\infty$

$= 0 - (-3) = 3$

$\therefore Var(X+1) = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$

$\therefore Var(X+1) = Var(X) = \frac{3}{4}$ ✱

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