LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

MELAKA (2006)
MATHEMATICS T (PAPER 1)

1. A line is drawn from the point $A(1, 2)$ to cut the lines $2y = 3x - 5$ and $x + y = 12$ at P and Q respectively. If P is the mid-point of AQ, find the coordinates of P and Q. [6]

2. A finance loan of RM 500 is arranged to be repaid in 24 months by equal monthly instalments. Interest, calculated monthly, is charged at 11% per annum on the balance of the loan. The first instalment is paid one month after the loan is made. Calculate the monthly payment. [6]

3. M is a matrix given by $M = \begin{pmatrix} 3 & 1 & -2 \\ 4 & 0 & -2 \\ 4 & -1 & -1 \end{pmatrix}$. Find M^2.

   Given that $M^3 = 2M^2 + M - 2I$, where I is an identity matrix. Show that $2M^{-1} + I - M^2$. Hence, find M^{-1}. [6]

4. Find the solution set of the inequality $\frac{2}{x-3} > \frac{3}{x-2}$. [4]

5. The function f is defined by $f(x) = 1 + 2\sqrt{x+2}$, for $x \geq -2$.

   (a) Find $f^{-1}(x)$ and state the domain of f^{-1}. [3]

   (b) State the line of reflection for the graph of f and the graph of f^{-1}. Hence, find the value of x for which $f(x) = f^{-1}(x)$. [4]

6. By using substitution $t = e^x$, find $\int \frac{1}{e^x - e^{-x}}\,dx$. [4]

7. The equation of a curve is $(x + y)^2 + 2(x - y)^2 = 24$. Show that the gradient of the curve at the point (x, y) may be expressed in the form $\frac{3x - y}{x - 3y}$.

   Hence, find the x–coordinates of the points on the curve where its tangent is parallel to the x–axis. [5]

8. The function g is defined as $g(x) = \begin{cases} ax + 2 & , -3 \leq x \leq 1 \\ x^2 + x + a & , 1 < x < 2 \\ 1 - \frac{1}{x} & , 2 \leq x \leq 5 \end{cases}$.

   (a) Show that g is continuous at $x = 1$. [3]

   (b) Find the value of a if g is continuous in the interval $[-3, 5]$. [2]

9. The polynomial $x^5 - 3x^4 + 2x^3 - 2x^2 + 3x + 1$ is denoted by $f(x)$.

   (a) Show that $(x - 1)$ and $(x + 1)$ are not factors of $f(x)$. [2]

   (b) By substituting $x = 1$ and $x = -1$ in the identity $f(x) \equiv (x^2 - 1).q(x) + ax + b$, where $q(x)$ is a polynomial and a and b are constants, find the remainder when $f(x)$ is divided by $(x^2 - 1)$. [3]

   (c) Find the remainder when $f(x)$ is divided by $(x^2 + 1)$. [3]

   (d) Find all the real roots of the equation $f(x) = 2x$. [6]

10. (a) Express $\frac{1}{(1+x)(2+x)}$ in partial fractions. [2]

    Hence, expand the function as a series of ascending powers of x up to and including the term in x^2. [4]

    (b) Show that the first four non-zero terms in the expansion of $\sqrt[n]{1 - \frac{1}{n}}$ in ascending powers of $\frac{1}{n}$ are . [5]

    By substituting n with a suitable value, use the first four non-zero terms of the series to estimate value of $\sqrt[10]{0.9}$, giving your answer correct to five decimal places. [3]

11. (a) Sketch the graph of $y = \frac{1-x}{x-2}$. State the equations of the asymptotes. [4]

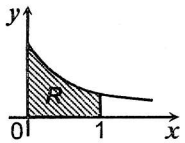
    (b) On separate axes, sketch the graph of $y = \left|\frac{1-x}{x-2}\right|$. [2]

    (c) On the same axes as part (a) above, sketch and label clearly the graph of $y = -x^3$.

    Hence, state the number of positive real roots, and the number of negative real roots of the equation, $x^4 - 2x^3 - x + 1 = 0$. [3]

    Starting with $x = 2$, use the Newton-Raphson method to find the largest positive root of the equation, $x^4 - 2x^3 - x + 1 = 0$, correct to one decimal place. [5]

12. (a) The diagram shows the region R bounded by the axes, the curve $y = \sqrt{(x^2+1)^3}$ and the line $x = 1$. By using the trapezium rule, with ordinates $x = 0$, $x = \frac{1}{2}$ and $x = 1$, estimate the volume of the solid formed when R is rotated completely about the x–axis. Give your answer correct to two significant figures.



    (b) Given that $f(x) = \begin{cases} x^2 & , 0 < x \leq 1 \\ 3 - 2x & , 1 < x \leq 3 \end{cases}$, and $f(x + 3) = f(x)$ for all values of x.

    Sketch $f(x)$, for the range $3 < x \leq 7$.

    Evaluate (i) $f(23)$ (ii) $\int_{-3}^{7} f(x)\,dx$. [10]

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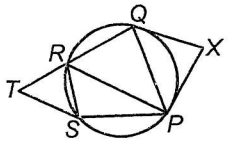
MATHEMATICS T (PAPER 2)

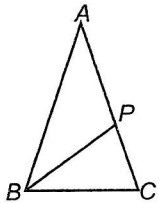
1. $PQRS$ is a cyclic quadrilateral in which $SP = PQ$.



   (a) If T is the point on QR produced such that ST is parallel to PR, prove that $RT = SR$. [4]

   (b) If the tangents at P and Q to the circle $PQRS$ meet at X, prove that $\angle PXQ = \angle SRT$. [3]

2. In the figure, $BC = BP = AP$ and the angles BAC, ABP, and BPC are each $\frac{\pi}{5}$ radians.



   Show that ΔABC and ΔBPC are similar. [3]

   Hence, deduce that $BC^2 = CP.CA$ and $\cos\frac{\pi}{5} = \frac{1}{4}(\sqrt{5}+1)$. [7]

3. (a) Prove that $(2\cos 3\theta + 2\cos 2\theta + 2\cos\theta + 1)\sin\frac{\theta}{2} = \sin\frac{7\theta}{2}$. [3]

   (b) Express $\cos 3\theta$ and $\cos 2\theta$ in terms of $\cos\theta$, deduce that
   $$(8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1)\sin\frac{\theta}{2} = \sin\frac{7\theta}{2}.$$
   Hence, deduce that $\cos\frac{2\pi}{7}$ is one of the roots of the equation
   $$8x^3 + 4x^2 - 4x - 1 = 0.$$ [6]

4. At 1700 hours, a pirate's ship Q is 25 nautical miles west of a police boat P. Q is sailing in the direction N 10o E at 20 knots and P is moving at 30 knots. Find to the nearest degree, the direction P should be travelling in order to intercept Q, and determine the time when the interception takes place. [7]

5. Prove that $\frac{d}{dx}\left(\sqrt{\frac{2+x}{2-x}}\right) = \frac{2}{\sqrt{(2-x)^3(2+x)}}$. [2]

   Given that $\frac{dy}{dx} = \frac{e^y}{\sqrt{(2-x)^3(2+x)}}$ and that $y = 0$ when $x = 1$, express y in terms of x. [4]

6. For all positive values of x, the gradient of normal of a curve at point (x, y) is $-\frac{x^2+x}{y}$.

   The point $(3, 6)$ lies on this curve. Find the equation of the curve in the form $y = f(x)$. [6]

7. A listed company bids for government projects to build highways. The person in charge estimates that the probability of winning the first project is 0.6, the probability of winning the second project is 0.4 and winning both projects is 0.25.

   (a) Find the probability that the company does not win any project. [2]

   (b) Find the probability that the company wins exactly one project. [1]

   (c) Given that the company does not win the first project, find the probability that he wins the second. [3]

   (d) Determine whether or not, winning the first contract and winning the second contract are independent events. [2]

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8. Whenever an amateur acrobat performs a certain routine, the probability that she will do so faultlessly is 0.85. Find correct to three decimal places, the probability that she will perform the routine faultlessly on at least four occasions out of six. [3]

9. The door frames provided by a housing contractor are of a standard height, 1.90 m. The heights of men are normally distributed with mean 1.75 m and standard deviation 0.08 m.

   (a) Find the proportion of men who are taller than the door frame. [2]

   (b) If the contractor wishes to alter the frame height, in such a way that forty nine men out of fifty are not taller than the frame height, find correct to two decimal places, the new height of the door frame. [3]

   (c) The door frames are to be used in a department store. It is known that women outnumber men in the ratio 15 : 1 in the store, and that the proportion of women taller than the door frame is 0.005. Find, correct to 4 decimal places, the proportion of people for whom a frame height of 1.90 m would be too low. [3]

10. (a) A random sample of 200 rubber trees yields the following information concerning their trunk diameters, in centimetres.

Minimum	Lower quartile	Median	Upper quartile	Maximum
13	27	32	35	42

   Use this data summary to draw a cumulative frequency curve. Hence determine the interpercentile range. [6]

   (b) The table shows the daily expenses (RM) of 80 college students.

Expenses (RM x)	Frequency
$20 \le x < 25$	5
$25 \le x < 30$	10
$30 \le x < 35$	17
$35 \le x < 40$	26
$40 \le x < 45$	18
$45 \le x < 50$	4

   Draw a histogram for the data.

   Use your histogram to determine the value of k given that the probability of the daily expenses greater than k is 0.1. [6]

11. A fairground game involves hitting a moving target with a gunshot. A round consists of three shots. Ten points are scored if a player hits the target at each shot, but the round is over if the player misses the three shots. Fiona has a constant probability of 0.8 of hitting the target at each shot and the shots are independent of one another. The random variable X is the score Fiona obtains in a round. Find

   (a) the probability distribution of X. [3]

   (b) the mean and the standard deviation of X. [5]

   A game consists of 2 rounds, find the probability that Fiona scores more points in round 2 than in round 1. [3]

   Given that $Y = \frac{X}{2} - 1$. Find the mean and the standard deviation of Y. [4]

12. The cumulative distribution function of X is given by $F(x) = \begin{cases} 0 & , x \le 0 \\ 1 - e^{-5x} & , x > 0 \end{cases}$

   Find the probability density function of X, hence find the mean of X. [4]

   If the continuous random variable Y is denoted by $Y = 3X + 2$. Find $P(Y \le y)$ and the probability density function of Y, [5]

1. Let $P(p, \frac{3p-5}{2})$, $Q(q, 12-q)$

$P = M_{AQ}$: $(p, \frac{3p-5}{2}) = (\frac{q+1}{2}, \frac{12-q+2}{2})$

$\therefore 2p = q+1$ and $3p = 19-q$

$\therefore 5p = 20 \Rightarrow p = 4 \quad \therefore q = 7$

$\therefore P(4, \frac{7}{2})$, $Q(7, 5)$ ✱

2. Let B_n = balance after n^{th} month

P = monthly instalment.

$B_1 = 500 + 500(\frac{0.11}{12}) - P$

$= d(500) - P$, (Let $d = \frac{1211}{1200}$)

$B_2 = dB_1 - P = d^2(500) - P(1+d)$

$B_3 = dB_2 - P = d^3(500) - P(1+d+d^2)$

since $B_{24} = 0$:

$d^{24}(500) = P(1+d+d^2+\dots+d^{23})$

$\therefore P = \frac{d^{24}(500)}{\frac{d^{24}-1}{d-1}} = RM23.30$ ✱

3. (a) $M^2 = \begin{pmatrix} 9+4-8 & 3+0+2 & -6-2+2 \\ 12+0-8 & 4+0+2 & -8+0+2 \\ 12-4-4 & 4+0+1 & -8+2+1 \end{pmatrix} = \begin{pmatrix} 5 & 5 & -6 \\ 4 & 6 & -6 \\ 4 & 5 & -5 \end{pmatrix}$ ✱

(b) $M^3 = 2M^2 + M - 2I$ —$(\times M^{-1})$

$M^2 = 2M + I - 2M^{-1}$

$\therefore 2M^{-1} = 2M + I - M^2$ ✱

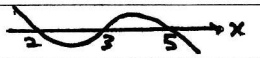
$\therefore M^{-1} = \frac{1}{2}\left[2\begin{pmatrix} 3 & 1 & -2 \\ 4 & 0 & -2 \\ 4 & -1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 5 & 5 & -6 \\ 4 & 6 & -6 \\ 4 & 5 & -5 \end{pmatrix}\right]$

$= \frac{1}{2}\begin{pmatrix} 2 & -3 & 2 \\ 4 & -5 & 2 \\ 4 & -7 & 4 \end{pmatrix}$ ✱

4. $\frac{2}{x-3} > \frac{3}{x-2} \Rightarrow 2(x-2)^2(x-3) > 3(x-3)^2(x-2)$

$(x-2)(x-3)[2(x-2) - 3(x-3)] > 0$

$\therefore (x-2)(x-3)(5-x) > 0$



$\therefore \{x : x < 2 \text{ or } 3 < x < 5\}$ ✱

5. (a) $f^{-1}(x) = y \Rightarrow x = f(y) = 1 + 2\sqrt{y+2}$

$\therefore (\frac{x-1}{2})^2 = y+2 \Rightarrow y = (\frac{x-1}{2})^2 - 2$

$\therefore f^{-1}(x) = \frac{1}{4}(x-1)^2 - 2$, $x \geq 1$ ($D_{f^{-1}}$) ✱

(b) Line of symmetry: $y = x$ ✱

For $f(x) = f^{-1}(x)$: $\frac{1}{4}(x^2 - 2x + 1) - 2 = x$

$\therefore x^2 - 6x - 7 = 0 \Rightarrow (x-7)(x+1) = 0$

$[\because x \geq 1]$: $x = 7$ ✱

6. $t = e^x$: $\frac{dt}{dx} = e^x = t$

$\int \frac{1}{e^x - e^{-x}} dx = \int \frac{t}{t^2-1}(\frac{dt}{t}) = \int \frac{1}{t^2-1} dt$

$= \frac{1}{2}\int [\frac{1}{t-1} - \frac{1}{t+1}] dt$

$= \frac{1}{2}[\ln(\frac{t-1}{t+1})] + c = \frac{1}{2}\ln[\frac{e^x-1}{e^x+1}] + c$ ✱

7. $(x+y)^2 + 2(x-y)^2 = 24$ —$(\frac{d}{dx})$

$\therefore 2(x+y)(1+y') + 4(x-y)(1-y') = 0$

$\therefore y'(2x+2y-4x+4y) + 2x+2y+4x-4y = 0$

$\therefore \frac{dy}{dx} = \frac{3x-y}{x-3y}$ ✱

tangent // x-axis: $\frac{dy}{dx} = 0 \Rightarrow y = 3x$

$\therefore (4x)^2 + 2(-2x)^2 = 24 \Rightarrow x = \pm 1$ ✱

8. (a) $g(1) = a(1) + 2 = a+2$

$\lim_{x \to 1^+} g(x) = 1 + 1 + a = a+2$

$\lim_{x \to 1^-} g(x) = a(1) + 2 = a+2$

$\therefore \lim_{x \to 1^-} g(x) = g(1) = \lim_{x \to 1^+} g(x) = a+2$

$\Rightarrow g$ is continuous at $x = 1$ ✱

(b) g continuous at $x = 2$: $\lim_{x \to 2^-} g(x) = g(2)$

$\therefore 4 + 2 + a = 1 - \frac{1}{2} \Rightarrow a = -\frac{11}{2}$ ✱

9. $f(x) = x^5 - 3x^4 + 2x^3 - 2x^2 + 3x + 1$

(a) $f(1) = 1 - 3 + 2 - 2 + 3 + 1 = 2 \quad (\neq 0)$

$f(-1) = -1 - 3 - 2 - 2 - 3 + 1 = -10 \ (\neq 0)$

$\therefore (x-1)$ and $(x+1)$ are not factors of $f(x)$ ✱

(b) $f(x) \equiv (x^2-1)q(x) + ax + b$

$f(1) = 2$: $2 = a + b$, $f(-1) = -10$: $-10 = -a + b$

$\therefore a = 6$ and $b = -4$

$\therefore$ Remainder $= 6x - 4$ ✱

(c)

$$\begin{array}{r|l} & x^3 - 3x^2 + x + 1 \\ \hline x^2+1 & x^5 - 3x^4 + 2x^3 - 2x^2 + 3x + 1 \\ & x^5 \qquad + x^3 \\ \hline & -3x^4 + x^3 - 2x^2 \\ & -3x^4 \qquad - 3x^2 \\ \hline & x^3 + x^2 + 3x \\ & x^3 \qquad + x \\ \hline & x^2 + 2x + 1 \\ & x^2 \qquad + 1 \\ \hline & 2x \rightarrow \text{Remainder} \end{array}$$

✱

(d) $f(x) = 2x$: $(x^2+1)(x^3 - 3x^2 + x + 1) = 0$

$[\because x \in \mathbb{R}]$: $(x-1)(x^2 - 2x - 1) = 0$

$x = 1$ or $\frac{1}{2}[2 \pm \sqrt{4+4}]$

$\therefore$ Root (real) $= 1, 1+\sqrt{2}, 1-\sqrt{2}$ ✱

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10.(a) $f(x) = \frac{1}{(1+x)(2+x)} = \frac{1}{1+x} - \frac{1}{2+x}$ *

$f(x) = (1+x)^{-1} - \frac{1}{2}(1+\frac{x}{2})^{-1}$

$= [1 - x + x^2 + \dots] - \frac{1}{2}[1 - \frac{x}{2} + \frac{x^2}{4} + \dots]$

$= \frac{1}{2} - \frac{3}{4}x + \frac{7}{8}x^2 + \dots$ *

(b) $(1-\frac{1}{n})^{\frac{1}{n}} = 1 + \frac{1}{n}(-\frac{1}{n}) + \frac{1}{2}(\frac{1}{n})(\frac{1}{n}-1)(-\frac{1}{n})^2$

$+ \frac{1}{6}(\frac{1}{n})(\frac{1}{n}-1)(\frac{1}{n}-2)(\frac{1}{n})^3 + \dots$

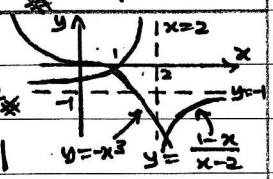
$= 1 - (\frac{1}{n})^2 + \frac{1}{2}[(\frac{1}{n})^4 - (\frac{1}{n})^3] + \frac{1}{6}[-2(\frac{1}{n})^4 + \dots] + \dots$

$= 1 - (\frac{1}{n})^2 - \frac{1}{2}(\frac{1}{n})^3 + \frac{1}{6}(\frac{1}{n})^4 + \dots$ *

$\underline{n=10}$: $(0.9)^{\frac{1}{10}} = 1 - 0.1^2 - \frac{1}{2}(0.1)^3 + \frac{1}{6}(0.1)^4 + \dots$

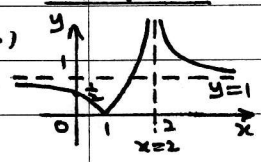
$= 0.98952$ (5dp) *

11.(a) $y = \frac{1-x}{x-2} = -1 - \frac{1}{x-2}$



asymptotes: $y=-1, x=2$ *

(b)



$y = \left|\frac{1-x}{x-2}\right|$

(c) $x^4 - 2x^3 - x + 1 = 0 \Rightarrow x^3(x-2) = x-1$

$\therefore -x^3 = \frac{1-x}{x-2} \Rightarrow$ No. of positive roots = 2 and no negative root. *

Let $f(x) = x^4 - 2x^3 - x + 1$: $f'(x) = 4x^3 - 6x^2 - 1$

By NRM: $x' = x - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$

$x_0 = 2$: $x_1 = 2.143 \approx 2.1$

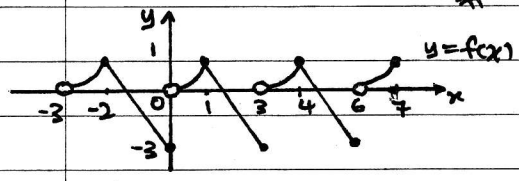
$x_2 = 2.119 \approx 2.1$

$\therefore$ root $= 2.1$ (1dp) *

12.(a) $V_x = \pi \int_0^1 (x^2+1)^{-3} dx$

$\approx \pi(\frac{1}{2})(\frac{1}{2})[1 + \frac{2}{(1.25)^3} + \frac{1}{8}] = 1.7$ (2s.f) *

(b)

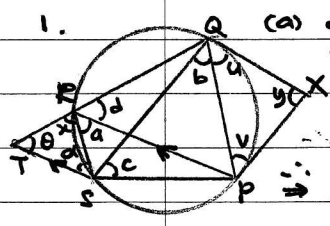


(i) $f(23) = f(2) = 3 - 2(2) = -1$

(ii) $\int_3^7 f(x)\,dx = 4\int_0^1 x^2 dx + 3[\frac{1}{2}(1)(\frac{1}{2}) - \frac{1}{2}(\frac{3}{2})(3)]$

$= \frac{4}{3}[x^3]_0^1 - 6 = \frac{4}{3}(1-0) - 6 = -\frac{14}{3}$ *

1.



(a) $d = a$ [Alternate ∠, ST // PR]

$a = b$ [∠ in the same segment]

$c = b$ [PQ = PR – ΔPQS isosceles]

$c = d$ [∠ in the same segment]

$\theta = d$ [corresponding ∠]

$\therefore \theta = d$

$\Rightarrow$ ΔRST is isosceles with RS = RT *

(b) $u = c$ [∠ in alternate segment]

$v = d$ [∠ in alternate segment]

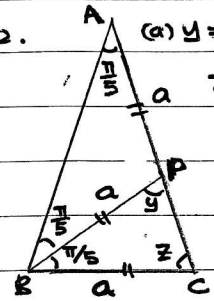
$u + v + y = 180°$ [Σ ∠ in Δ]

$\therefore y = 180° - c - d = 180° - a - d$

$= x$ [Σ angles on QRT]

$\therefore \angle PXQ = \angle SRT$ *

2.



(a) $y = 2(\frac{\pi}{5}) = \frac{2\pi}{5}$ [ext. ∠ of Δ]

$z = y = \frac{2\pi}{5}$ [BC = BP]

$\therefore \angle BAC = \angle CBP = \frac{\pi}{5}$

$\angle ABC = \angle ACB = z = y = \frac{2\pi}{5}$

$\therefore \Delta ABC \sim \Delta BPC$ *

(b) (i) $\frac{BC}{PC} = \frac{AC}{BC} \Rightarrow BC^2 = PC.AC$ *

(ii) $AB = AC = 2[a\cos\frac{\pi}{5}] = 2a\cos\frac{\pi}{5}$

$\therefore \underline{BC^2 = PC.AC}$: $a^2 = [2a\cos\frac{\pi}{5} - a][2a\cos\frac{\pi}{5}]$

$\therefore 4\cos^2\frac{\pi}{5} - 2\cos\frac{\pi}{5} - 1 = 0$

$\therefore \cos\frac{\pi}{5} = \frac{1}{8}[2 \pm \sqrt{4+16}]$

$\therefore \cos\frac{\pi}{5} = \frac{1}{4}[1+\sqrt{5}]$, $(\because \cos\frac{\pi}{5} > 0)$ *

3.(a) $[2\cos3\theta + 2\cos2\theta + 2\cos\theta + 1]\sin\frac{\theta}{2}$

$= (\sin\frac{7\theta}{2} - \sin\frac{5\theta}{2}) + (\sin\frac{5\theta}{2} - \sin\frac{3\theta}{2})$

$+ [\sin\frac{3\theta}{2} - \sin\frac{\theta}{2}] + \sin\frac{\theta}{2} = \sin\frac{7\theta}{2}$ *

(b) $\cos3\theta = 4\cos^3\theta - 3\cos\theta$, $\cos2\theta = 2\cos^2\theta - 1$ *

$[8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1]\sin\frac{\theta}{2}$

$= [2(\cos3\theta + 3\cos\theta) + 2(\cos2\theta + 1) - 4\cos\theta - 1]\sin\frac{\theta}{2}$

$= [2\cos3\theta + 2\cos2\theta + 2\cos\theta + 1]\sin\frac{\theta}{2} = \sin\frac{7\theta}{2}$ *

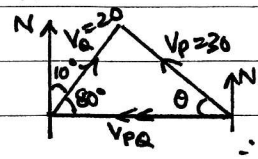
$\underline{\theta = \frac{2\pi}{7}}$: $\sin\frac{7\theta}{2} = \sin\pi = 0$ $[\because \sin\frac{2\pi}{7} \neq 0]$

$\theta = \frac{2\pi}{7}$ is a solution for

$8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1 = 0$ [let $x = \cos\theta$]

$\therefore \cos\frac{2\pi}{7}$ is a root for $8x^3 + 4x^2 - 4x - 1 = 0$ *

4.



$\frac{\sin\theta}{20} = \frac{\sin80°}{30} \Rightarrow \theta = 41°$

$\therefore$ Direction of P = N49°W *

$\frac{|V_{PQ}|}{\sin59°} = \frac{30}{\sin80°}$

$\therefore |V_{PQ}| = 26.11$

$\therefore$ time taken $= \frac{25}{26.11}$ h = 57 minutes

$\therefore$ Time happen = 1757 *

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5. Let $u = \sqrt{\frac{2+x}{2-x}}$: $u^2 = \frac{4}{2-x} - 1$

$\therefore 2u\frac{du}{dx} = \frac{4}{(2-x)^2} \Rightarrow \frac{du}{dx} = \frac{2}{(2-x)^2}\sqrt{\frac{2-x}{2+x}}$

$\therefore \frac{du}{dx} = 2/(2-x)^{3/2}(2+x)^{1/2}$ *

$\frac{dy}{dx} = e^y/(2-x)^{3/2}(2+x)^{1/2}$

$\therefore \int 2e^{-y}\,dy = \int \left(\frac{dy}{dx}\right)dx$

$\therefore -2e^{-y} = u + c = \sqrt{\frac{2+x}{2-x}} + c$

$x=1$: $-2 = \sqrt{3} + c$

$\therefore -2e^{-y} = \sqrt{\frac{2+x}{2-x}} - 2 - \sqrt{3}$

$\therefore y = -\ln\left[1 + \frac{\sqrt{3}}{2} - \frac{1}{2}\sqrt{\frac{2+x}{2-x}}\right]$ *

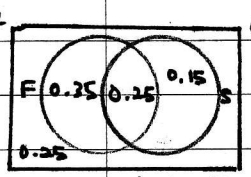
6. $\frac{dy}{dx} = \frac{y}{x^2+x}$: $\int \frac{1}{y}\,dy = \int\left[\frac{1}{x} - \frac{1}{x+1}\right]dx$

$\ln y = \ln\left(\frac{x}{x+1}\right) + c$

$(3,6)$: $\ln 6 = \ln\frac{3}{4} + c \Rightarrow c = \ln 8$

$\therefore \ln y = \ln\left(\frac{x}{x+1}\right) + \ln 8 \Rightarrow y = \frac{8x}{x+1}$ *

7.



(a) $P(F \cup S)' = 0.25$ *

(b) $P(\text{only one}) = 0.35 + 0.15 = 0.5$ *

(c) $P(S|F') = \frac{P(S \cap F')}{P(F')} = \frac{0.15}{0.4} = \frac{3}{8}$ *

(d) $P(F).P(S) = (0.6)(0.4) = 0.24$ $[\neq P(F \cap S)]$

$\therefore$ F and S are not independent *

8. Prob. $= {}^6C_4(0.85)^4(0.15)^2 + {}^6C_5(0.85)^5(0.15) + (0.85)^6 = 0.953$ (3dp) *

9. Let X = height of man

$X \sim N(1.75, 0.08^2)$

(a) $P(X > 1.90) = P\left(Z > \frac{1.90-1.75}{0.08}\right) = 0.0304$ *

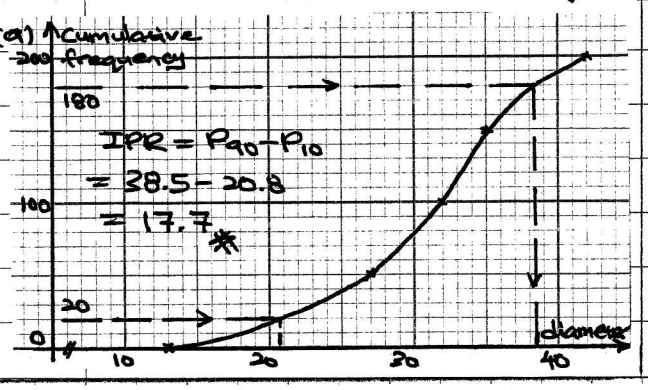
(b) $P(X > k) = \frac{1}{50} \Rightarrow \frac{k-1.75}{0.08} = z_{0.02}$

$\therefore k - 1.75 = 0.08(2.054) \Rightarrow k = 1.91$ (2dp) *

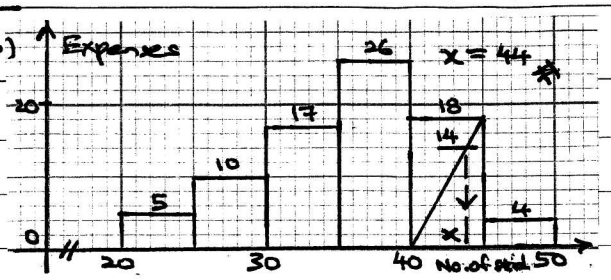
(c) $P(\text{people} > 1.90) = \frac{1}{16}(0.0304) + \frac{15}{16}(0.005)$

$= 0.0066$ (4dp) *

10.(a)



(b)



11(a) $P(T=t) = {}^3C_t(0.8)^t(0.2)^{3-t}$, $T \sim B(3, 0.8)$

x	0	10	20	30
$P(X=x)$	0.008	0.096	0.384	0.512

$X = 10T$ *

(b) $E(X) = 10(3)(0.8) = 24$ *

$S_X = 10\sqrt{3(0.8)(0.2)} = 4\sqrt{3}$ *

$P(X_2 > X_1) = (0.992)(0.008) + 0.896(0.096) + 0.512(0.384) = 0.29056$ *

$Y = \frac{X}{2} - 1$: $E(Y) = \frac{1}{2}(24) - 1 = 11$ *

$S_Y = \frac{1}{2}(4\sqrt{3}) = 2\sqrt{3}$ *

12. $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} 5e^{-5x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ *

$E(X) = \int_0^\infty 5xe^{-5x}\,dx$

$= \left[x(-e^{-5x})\right]_0^\infty + \int_0^\infty e^{-5x}\,dx$

$= (0-0) - \frac{1}{5}\left[e^{-5x}\right]_0^\infty$

$= -\frac{1}{5}[0-1] = \frac{1}{5}$ *

(a) $P(Y \leq y) = P(3X+2 \leq y)$

$= P\left(X \leq \frac{y-2}{3}\right)$

$= 1 - e^{\frac{5}{3}(2-y)}$, for $\frac{y-2}{3} \geq 0$

$= 1 - e^{\frac{5}{3}(2-y)}$, for $y \geq 2$ *

(b) $G(y) = \begin{cases} 1 - e^{\frac{5}{3}(2-y)}, & y \geq 2 \\ 0, & y < 2 \end{cases}$

$g(y) = \frac{d}{dy}[G(y)] = \begin{cases} \frac{5}{3}e^{\frac{5}{3}(2-y)}, & y \geq 2 \\ 0, & y < 2 \end{cases}$ *

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