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M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

DAERAH SEGAMAT, JOHOR (2006)
MATHEMATICS T (PAPER 1)

1. Solve the equation $\log_2 x = \log_x 8$, giving your answer correct to 3 significant figures. [5]

2. Find the perpendicular distance between the parallel lines

   $3x + 5y + 2 = 0$ and $3x + 5y + 10 = 0$. [5]

3. Find the volume of the solid generated when the area bounded by the curve $y = x^2$ and $y^2 = 27x$ is rotated completely about the x–axis. [4]

4. The functions f and g are given as $f : x \to \sqrt{3-x}$, $x \le k$ and $g : x \to e^x$, $x \ge 1$.
   If $k = 3$, show that gf does not exist. [3]

   Find the largest possible value k such that gf exists.
   Hence, define gf in a similar form and find the range. [4]

5. Show that $x + 1$ is a factor of $f(x) = x^3 - 5x^2 + 3x + 9$, then factorise $f(x)$ completely. [3]

   (a) Sketch the graph of $y = f(x)$, and use it to find the solution set of the inequality $x^3 - 5x^2 + 3x + 9 > 0$. [4]

   (b) Express $\dfrac{2x}{x^3 - 5x^2 + 3x + 9}$ in partial fractions. [3]

6. (a) Find $\int x(x^2 - 3)^3 \, dx$. [3]

   (b) Evaluate $\int_0^{\frac{\pi}{4}} \sin 2\theta \, d\theta$. [3]

7. Find the expansion of $\sqrt{\dfrac{1+x^2}{1-x}}$ in the ascending powers of x until the terms in x^3.

   State the range of values of x for the expansion to be valid. [7]

   Suggest a suitable value of x for which $\sqrt{\dfrac{13}{10}}$ can be estimated by using above expansion. Hence find the approximate value of $\sqrt{\dfrac{13}{10}}$ correct to four decimal places. [4]

8. Find the approximate value for $\int_0^2 \dfrac{1}{e^x} \, dx$, correct to two decimal places by using the trapezium rule with five ordinates. [4]

   Calculate the exact value of $\int_0^2 \dfrac{1}{e^x} \, dx$, correct to two decimal places. [3]

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9. Function f is defined by $f(x) = \begin{cases} (2-x)^2 & , \quad x \le 1 \\ 2 - \dfrac{a}{x} & , 1 < x \le 2 \\ 3x & , 2 < x \end{cases}$, where a is a constant.

   If f is continuous at $x = 1$, determine the value of a and the existence of $\lim\limits_{x \to 2} f(x)$. [5]

   Sketch the graph of function f. [4]

10. Matrices A and B are given as $A = \begin{pmatrix} 2 & 1 & -2 \\ -1 & 1 & -3 \\ -3 & -2 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}$.

    Find AB and deduce A^{-1}. [4]

    Hence, express the following simultaneous equations as a matrix equation and solve it, $2x + y - 2z = 3$, $-2x + 2y - 6z = -14$ and $-3x - 2y + 2z = -5$. [5]

11. Find the coordinates of the point where the curve $y = \frac{1}{2}x^3 - x^2 - 2x$ intersects the x–axis. Find also the local minimum and local maximum points, as well as the point of inflexion. [10]

    Hence, sketch the curve. [2]

    Find the possible values of k such that the equation $\frac{1}{2}x^3 - x^2 - 2x = k$ has a repeated root, and state this root. [4]

12. (a) The first three terms of an arithmetic series is $-\frac{7}{6}, -\frac{2}{3}, -\frac{1}{6}$.

    Show that the sum of first n terms, S_n, of this series is $\frac{n}{12}(3n - 17)$. [3]

    Hence, find the value of n for which S_n is minimum. [2]

    (b) The first term and common ratio of a geometric series are 2 and r respectively.
    If S_n is the sum of first n terms and $6S_3 = 7S_2$, find both possible values of r. [3]

    For each value of r, find the sum to infinity for the corresponding series. [3]

DAERAH SEGAMAT, JOHOR (2006)
MATHEMATICS T (PAPER 2)

1. Find the set of values of x in the range $0^\circ \le x \le 180^\circ$ that satisfies the equation

   $\sin 2x + \sin 3x + \sin 5x = 0$ [5]

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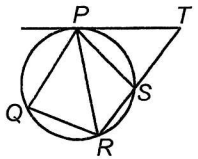
2. The points P, Q, R and S are on the circumference of a circle, such that $\angle PQR = 80^\circ$ and $\angle RPS = 30^\circ$, as shown in the above diagram. The tangent to the circle at P and the chord RS which is produced, meet at T.



(a) Show that $PR = PT$ [3]

(b) Show that the length of the chord RS is same as the radius of the circle. [3]

3. The table shows the number of women based on the number of their children according to result of a survey conducted on 50 000 married women in the country.

No. of children	0	1	2	3	4	5	6
No. of women	5011	8698	11633	19418	4848	307	85

(a) Determine the percentage of women whose number of children are within the range of one standard deviation from the mean. [7]

(b) Let X represents the number of children of a woman that is chosen randomly. Tabulate the probability distribution of X with each probability given correct to three decimal places. [3]

(c) If five women are chosen randomly, find the probability that three of them have more than two children. [5]

4. A large quantity of shoes from brand B is inspected before been distributed to sales outlets. It is found that 1% of the left side shoes are defective while 2% of the right side shoes are defective. By assuming that the defects occur independently and using suitable approximation, calculate the probability that in a random sample of 50 pairs of shoes,

(a) there are two pairs of shoes with defects on their left side. [4]

(b) there is a pair of shoes with defects on both sides [4]

5. Continuous random variable X has the following probability density function

$$f(x) = \begin{cases} \frac{1}{8} & ,1 \le x \le 9 \\ 0 & ,\text{otherwise} \end{cases}$$

Continuous random variable Y is defined by $Y = \ln X$.

(a) Show that $P(Y \le y) = \frac{1}{8}(e^y - 1)$, $0 \le y \le \ln 9$. [4]

(b) Find and sketch the cumulative distribution function of Y. [4]

(c) Find the probability density function of Y. [3]

(d) Show that $E(Y) = \frac{9}{4}\ln 3 - 1$. [4]

6. An aircraft A which is located 500 km due south of an airport B, is flying at a constant height. The velocity of the aircraft relative to the air has a magnitude of 300 km h^{-1} and in the direction 060°. The wind is blowing from the direction 150° with a speed of 100 km h^{-1}. Find the actual velocity of the aircraft. [3]

If the aircraft maintains its speed but changes course in order to land at B, find the course A should steer and the time taken to reach B. [4]

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7. Show that by substituting $y = \frac{v}{x^2}$, the differential equation $\frac{dy}{dx} + y^2 = -\frac{2y}{x}$ may be reduced to $\frac{dv}{dx} = -\frac{v^2}{x^2}$. Hence, find y in terms of x, given that $y = 1$ when $x = 1$. [8]

8. A and B are two events with $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{3}$, and $P(A \cap B) = \frac{1}{2}$.
Evaluate $P(A' \cap B')$ and $P(A' \cup B')$. [4]

9. The table shows the probability distribution of X where $0 < a < \frac{1}{3}$. Show that $E(X) = 3 - 4a$ and find $Var(X)$ in terms of a. [4]

x	1	2	3
$P(X = x)$	a	$2a$	$1 - 3a$

The random variable S is the sum of n independent values of X.
Write down $E(S)$ and $Var(S)$ in terms of a and n. [2]

The random variable T is defined by $T = p + qS$. The values of p and q are such that $E(T) = a$ for all a in the interval $0 < a < \frac{1}{3}$. Find the values p and q. [4]

10. The horizontal base of a pyramid is an equilateral triangle ABC with each side 10 m long. The vertex V of the pyramid is at height of $\frac{10\sqrt{6}}{3}$ m above base. Find VB. [4]

If the point P lies on VB such that $VP = \frac{1}{5}VB$, find AP. [3]

11. The assembly of the student of a school begins 15 minutes after 8 o'clock every school day. A male student changes his shoes once he reaches the school. The time taken in minutes, from the time he arrives at school, to change his shoes, can be taken to have a normal distribution with mean 2 and standard deviation 1.5. The time taken in minutes after 8 o'clock he arrives at school can be taken to have a normal distribution with mean 10 and standard deviation 2. The time taken to change his shoes can be assumed to be independent of the time he arrives at the school.

(a) Show that on a day selected at random, the probability that the student has changed his shoes before assembly begins is 0.885.(correct to 3 significant figures). [3]

(b) Find the probability that, for 10 days selected at random, the student changes his shoes before assembly starts on less than 9 days out of the 10 days. [2]

(c) By using a suitable approximation, find the probability that, for 100 days selected randomly, the student changes his shoes before assembly starts is less than 95 days out of the 100 days. [4]

12. A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random, noting its colour, and replacing it together with an additional ball of the same colour. Given that three trials are made, find the probability that

(a) at least one blue ball is drawn, (b) exactly one blue ball is drawn. [4]

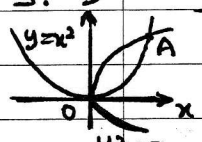
Given that all three balls drawn are of the same colour, find the probability that they are all red. [2]

1. $\log_2 x = \log_x 8$: $\log_2 x = \dfrac{3}{\log_2 x}$

$\therefore \log_2 x = \pm\sqrt{3} \Rightarrow x = 2^{\pm\sqrt{3}}$

$\therefore x = 3.32$ or 0.301 (3sf) ✱

2. distance $= \dfrac{|0+0+10| - 10+0+2|}{\sqrt{9+25}} = \dfrac{8}{\sqrt{34}}$ ✱

3. 

For A: $(x^2)^2 = 27x$

$\therefore x(x^3-27)=0 \Rightarrow x=0$ or 3

$\therefore x_A = 3$

$V_x = \pi\int_0^3 [27x - x^4]\,dx$

$= \pi\left[\frac{27}{2}x^2 - \frac{1}{5}x^5\right]_0^3 = \pi\left[\frac{243}{2} - \frac{243}{5}\right] = 72.9\pi$ ✱

4. $k=3$: $R_f: x \geq 0$, $D_g: x \geq 1$ } $R_f \not\subset D_g$ $\therefore$ gf does not exist ✱

For gf exist: $\sqrt{3-x} \geq 1 \Rightarrow x \leq 2$

$\therefore k_{max} = 2$ ✱

$gf(x) = g(\sqrt{3-x}) = e^{\sqrt{3-x}}$, $x \leq 2$ ✱

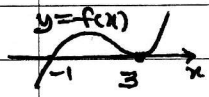
$\therefore R_{gf} = \{x : x \geq e\}$ ✱

5. $f(x) = x^3 - 5x^2 + 3x + 9$

$f(-1) = -1 - 5 - 3 + 9 = 0$

$\therefore (x+1)$ is a factor of $f(x)$ ✱

$f(x) = (x+1)(x^2 - 6x + 9) = (x+1)(x-3)^2$ ✱

(a) 

For $f(x) > 0$: $\{x : -1 < x < 3$ or $3 < x\}$ ✱

(b) $\dfrac{2x}{f(x)} \equiv \dfrac{A}{x+1} + \dfrac{B}{(x-3)} + \dfrac{C}{(x-3)^2}$

$\equiv \dfrac{A(x-3)^2 + B(x+1)(x-3) + C(x+1)}{f(x)}$

$\therefore 2x \equiv A(x-3)^2 + B(x+1)(x-3) + C(x+1)$

$x=-1$: $-2 = A(16) \Rightarrow A = -\frac{1}{8}$

$x=3$: $6 = C(4) \Rightarrow C = \frac{3}{2}$

(x^2): $0 = A + B \Rightarrow B = \frac{1}{8}$

$\therefore \dfrac{2x}{f(x)} = \dfrac{1}{8}\left[\dfrac{1}{x-3} + \dfrac{12}{(x-3)^2} - \dfrac{1}{x+1}\right]$ ✱

6(a) $u = x^2 - 3$: $\dfrac{du}{dx} = 2x$

$\int x(x^2-3)^3\,dx = \int 2u^3\,du = \frac{1}{2}u^4 + c$

$= \frac{1}{4}(x^2-3)^4 + c$ ✱

(b) $\int_0^{\pi/4} \sin 2\theta\, d\theta = -\frac{1}{2}[\cos 2\theta]_0^{\pi/4}$

$= -\frac{1}{2}[0-1] = \frac{1}{2}$ ✱

7. $f(x) = \sqrt{\dfrac{1+x^2}{1-x}} = [1 + \frac{1}{2}(x^2) + \ldots]$

$[1 + (-\frac{1}{2})(-x) + \frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-x)^2 + \frac{1}{6}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-x)^3 + \ldots$

$= (1 + \frac{1}{2}x^2 + \ldots)[1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots]$

$= 1 + \frac{1}{2}x + \frac{7}{8}x^2 + \frac{9}{16}x^3 + \ldots$ ✱

For exp. valid: $|x^2| < 1$ and $|x| < 1$

$\therefore \{x : -1 < x < 1\}$ ✱

$x = \frac{1}{5}$: $f(\frac{1}{5}) = \sqrt{\frac{26}{25} / \frac{4}{5}} = \sqrt{\frac{13}{10}}$

$\therefore \sqrt{\frac{13}{10}} = 1 + \frac{1}{2}(0.2) + \frac{7}{8}(0.2)^2 + \frac{9}{16}(0.2)^3 + \ldots$

$= 1.1395$ (4dp) ✱

8. $\int_0^2 \frac{1}{e^x}\,dx \approx \frac{1}{2}(\frac{1}{2})[1 + 2(e^{-0.5} + e^{-1} + e^{-1.5}) + e^{-2}] = 0.88$ (2dp) ✱

$\int_0^2 \frac{1}{e^x}\,dx = [-e^{-x}]_0^2 = 1 - e^{-2}$

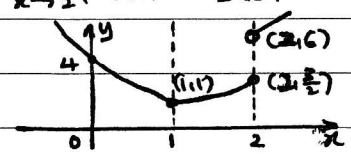
$= 0.86$ (2dp) ✱

9. $f(1) = \lim_{x\to 1^+} f(x)$: $1 = 2 - a \Rightarrow a = 1$ ✱

$\lim_{x\to 2^-} f(x) = 2 - \frac{1}{2} = \frac{3}{2}$

$\lim_{x\to 2^+} f(x) = 3(2) = 6$

} $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$ $\therefore \lim_{x\to 2} f(x)$ does not exist ✱



10. $AB = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$ ✱

$\therefore A^{-1} = \frac{1}{7}B = \frac{1}{7}\begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}$ ✱

$2x + y - 3z = 3$
$-2x + 2y - 6z = -14$
$-3x - 2y + 2z = -5$

} $2x + y - 3z = 3$
$-x + y - 3z = -7$
$-3x - 2y + 2z = -5$

$\therefore A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -7 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ -7 \\ -5 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \Rightarrow x = 3, y = -1, z = 1$ ✱

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11. $y=\frac{1}{2}x^3-x^2-2x$

$y'=\frac{3}{2}x^2-2x-2$, $y''=3x-2$

For $y'=0$: $3x^2-4x-4=0$

$\therefore (3x+2)(x-2)=0 \Rightarrow x=-\frac{2}{3}$ or 2

$x=2$: $y''=6-2=4\ (>0)$ $\therefore A(2,-4)$ min

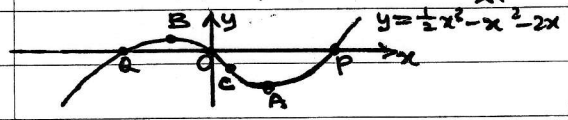
$x=-\frac{2}{3}$: $y''=-2-2=-4\ (<0)$ $\therefore B(-\frac{2}{3},\frac{20}{27})$ max

For $y''=0$: $x=\frac{2}{3}$ $\therefore C(\frac{2}{3},\frac{-44}{27})$ inf

For $y=0$: $x(x^2-2x-4)=0$.

$\therefore x=0$ or $1\pm\sqrt{5}$

$\therefore (0,0), P(1+\sqrt{5},0), Q(1-\sqrt{5},0)$



For $\frac{1}{2}x^3-x^2-2x=k$ to have repeated roots $\Rightarrow y=k$ is tangent to the curve

$\therefore k=-4$ or $\frac{20}{27}$

$k=-4$: repeated root $=2$

$k=\frac{20}{27}$: repeated root $=-\frac{2}{3}$

12(a) $S_n=\frac{n}{2}\left[2(-\frac{7}{6})+(n-1)(\frac{1}{2})\right]$

$=\frac{n}{12}[3n-17]$ —(Quadratic)

$\frac{n}{12}(3n-17)$ min at $n=\frac{17}{6}=2\frac{5}{6}$

$[\because n\in\mathbb{N}]$: (S_n) min. when $n=3$

(b) $6S_3=7S_2$: $6[2+2r+2r^2]=7[2+2r]$

$\therefore 6r^2-r-1=0 \Rightarrow (3r+1)(2r-1)=0$

$\therefore r=-\frac{1}{3}$ or $\frac{1}{2}$

$r=\frac{1}{2}$: $S_\infty=\frac{2}{1-\frac{1}{2}}=4$

$r=-\frac{1}{3}$: $S_\infty=2/1+\frac{1}{3}=\frac{3}{2}$

1. $\sin 2x+\sin 3x+\sin 5x=0$

$\therefore 2\sin x\cos x+2\sin 4x\cos x=0$

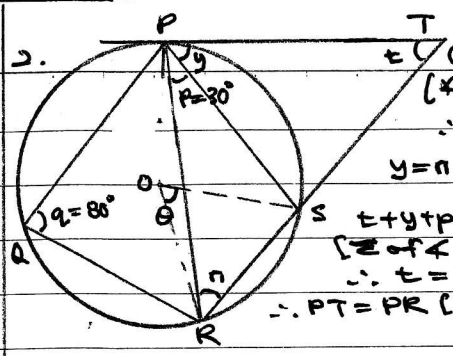
$\therefore \cos x=0$ or $2\sin\frac{5x}{2}\cos\frac{3x}{2}=0$

$\therefore x=90^\circ$ or $\sin\frac{5x}{2}=0$ or $\cos\frac{3x}{2}=0$

$\therefore x=90^\circ$ or $\frac{5x}{2}=0,180,360^\circ$ or $\frac{3x}{2}=90^\circ,270^\circ$

$\therefore x=90^\circ, 0^\circ, 72^\circ, 144^\circ, 60^\circ, 180^\circ$

2.



(a) $y+p=80^\circ$ [∠ of alternate segment]

$\therefore y=50^\circ$

$y=n=50^\circ$ [Angles of alternate segment]

$t+y+p+n=180^\circ$ [Σ of ∠ in Δ]

$\therefore t=50^\circ$ $(=n)$

$\therefore PT=PR$ [isosceles Δ]

(b) $\theta=2p=60^\circ$ [Angle at centre]

$\angle ORS=\angle OSR$ [ΔORS is isosceles]

$\angle ORS+\angle OSR+\theta=180^\circ$ [Σ of ∠ in Δ]

$\therefore \angle ORS=\angle OSR=\theta=60^\circ$

$\therefore$ ΔORS is equilateral $\Rightarrow RS=$ radius

3.(a) $n=50000$

$\bar{x}=\frac{\Sigma xf}{n}=\frac{111655}{50000}=2.233$

$s=\left[\frac{318295}{50000}-\left(\frac{111655}{50000}\right)^2\right]^{1/2}=1.174$

Range $=(\bar{x}-s,\bar{x}+s)=(1.059, 3.407)$

$\therefore$ Percentage $=\frac{11633+19418}{50000}\times 100\%$

$=62.102\%$

(b)

x	0	1	2	3	4	5	6
$P(X=x)$	0.100	0.174	0.233	0.388	0.097	0.006	0.02

(c) $P(X>2)=0.493$

Prob. $={}^5C_3(0.493)^3(0.507)^2=0.308$

4. X = no of left defective shoes

Y = no. of both side defective shoes.

(a) $X\sim B(50, 0.01) \rightarrow X\sim P_o(0.5)$

$P(X=2)=e^{-0.5}\left[\frac{0.5^2}{2!}\right]=0.0758$

(b) $Y\sim B(50, 0.01(0.02)) \rightarrow Y\sim P_o(0.01)$

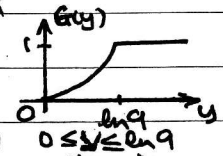
$P(Y=1)=e^{-0.01}\left[\frac{0.01}{1!}\right]=0.0099$

5(a) $P(Y\le y)=P(\ln X\le y)=P(X\le e^y)$

$=\int_0^{e^y}\frac{1}{8}\,dy$, for $1\le e^y\le 9$

$=\frac{1}{8}[e^y-1]$, for $0\le y\le \ln 9$

(b) $G(y)=\begin{cases}0 & y<0\\ \frac{1}{8}(e^y-1) & 0\le y\le \ln 9\\ 1 & \ln 9<y\end{cases}$



(c) $g(y)=\frac{d}{dy}[G(y)]=\begin{cases}\frac{1}{8}(e^y) & 0\le y\le \ln 9\\ 0 & \text{otherwise}\end{cases}$

(d) $E(Y)=\frac{1}{8}\int_0^{\ln 9} ye^y\,dy=\frac{1}{8}[ye^y]_0^{\ln 9}-\int_0^{\ln 9}\frac{1}{8}e^y\,dy$

$=\frac{1}{8}[9\ln 9-0]-\frac{1}{8}[e^y]_0^{\ln 9}=\frac{9}{4}\ln 3-1$

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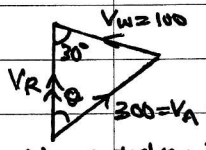
6. $V_R = 300\begin{pmatrix}\sin 60^\circ\\ \cos 60^\circ\end{pmatrix} + 100\begin{pmatrix}-\sin 30^\circ\\ \cos 30^\circ\end{pmatrix} = \begin{pmatrix}209.8\\ 236.6\end{pmatrix}$

$= \sqrt{209.8^2 + 236.6^2}$ kmh^{-1}, to N$\tan^{-1}\left(\frac{209.8}{236.6}\right)$E

$= 316.22$ kmh^{-1}, to N41.6°E



$\frac{\sin\theta}{100} = \frac{\sin 30^\circ}{300} \Rightarrow \theta = 9.6^\circ$

$\therefore$ Direction of A = N9.6°E

time taken $= \frac{500}{300\cos 9.6 + 100\cos 30^\circ} = 1.31$h

7. $y = \frac{v}{x^2} \Rightarrow x^2 y = v$

$\therefore x^2\frac{dy}{dx} + 2xy = \frac{dv}{dx}$

$\therefore \frac{dv}{dx} = x^2\left[-\frac{2y}{x} - y^2\right] + 2xy$

$\therefore \frac{dv}{dx} = x^2\left[-\left(\frac{v^2}{x^4}\right)\right] = -\frac{v^2}{x^2}$

$\int -\frac{1}{v^2}\,dv = \int \frac{1}{x^2}\,dx \Rightarrow \frac{1}{v} = -\frac{1}{x} + c$

$\therefore \frac{1}{x^2 y} = \frac{-1}{x} + c$

(1,1): $1 = -1 + c \Rightarrow c = 2$

$\therefore \frac{1}{y} = -x + 2x^2 \Rightarrow y = \frac{1}{2x^2 - x}$

8. (a) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - \left[\frac{3}{4} + \frac{2}{3} - \frac{1}{2}\right] = \frac{1}{12}$

(b) $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

$= 1 - \frac{1}{2} = \frac{1}{2}$

9. $E(X) = \Sigma x \cdot P(X = x)$

$= 1(a) + 2(2a) + 3(1 - 3a) = 3 - 4a$

$E(X^2) = 1^2(a) + 2^2(2a) + 3^2(1 - 3a) = 9 - 18a$

$\therefore Var(X) = (9 - 18a) - (3 - 4a)^2 = 6a - 16a^2$

$S = X_1 + X_2 + \ldots + X_n$:

$E(S) = n \cdot E(X) = n(3 - 4a)$

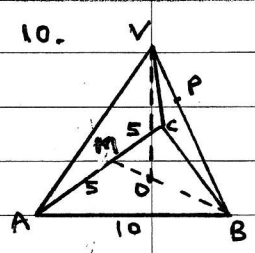
$Var(S) = n\,Var(X) = 2an(3 - 8a)$

$E(T) = a$: $p + qn(3 - 4a) = a$, $\forall a \in (0, \frac{1}{2})$

$\therefore [a^0]$: $p + 3qn = 0$ $[a]$: $-4qn = 1$

$\therefore p = -3\left(-\frac{1}{4}\right) = \frac{3}{4}$, $q = -\frac{1}{4n}$

10.



$BO = \frac{2}{3}\left[\sqrt{10^2 - 5^2}\right] = \frac{10}{3}\sqrt{3}$

$VB = \left[\frac{100}{9}(3) + \frac{100}{9}(6)\right]^{1/2} = 10$m

$VP = \frac{1}{5}VB = 2$, $VA = VB = 10$

$\therefore AP^2 = 10^2 + 2^2 - 2(10)(2)\left(\frac{1}{2}\right)$

$\therefore AP = \sqrt{84} = 2\sqrt{21}$ m

11. X = time taken to change shoes

Y = time taken to arrive after 8am

$X \sim N(2, 1.5^2)$, $Y \sim N(10, 2^2)$

(a) $P(X + Y < 15)$, $X + Y \sim N(2 + 10, 1.5^2 + 2^2)$

$= P\left(Z < \frac{15 - 12}{\sqrt{6.25}}\right) = 0.885$ (3 s.f.)

(b) Prob. $= 1 - \left[{}^{10}C_9(0.885)^9(0.115) + (0.885)^{10}\right]$

$= 0.3222$

(c) Let T = no. of days, shoes change before ass.

$T \sim B(100, 0.885)$

$\rightarrow T \sim N(88.5, 10.1775)$

$P(T < 95) \rightarrow P(T < 94.5)$

$= P\left(Z < \frac{94.5 - 88.5}{\sqrt{10.1775}}\right) = 0.9700$

12. (a) $P(\geq 1 \text{ blue}) = 1 - P(3 \text{ red})$

$= 1 - \frac{1}{3}\left(\frac{3}{4}\right)\left(\frac{3}{5}\right) = \frac{9}{10}$

(b) $P(1 \text{ blue}) = P(BRR) + P(RBR) + P(RRB)$

$= \frac{2}{3}\left(\frac{1}{4}\right)\left(\frac{3}{5}\right) + \frac{1}{3}\left(\frac{3}{4}\right)\left(\frac{3}{5}\right) + \frac{1}{3}\left(\frac{3}{4}\right)\left(\frac{2}{5}\right) = \frac{1}{5}$

$P(3R \mid \text{all same colour})$

$= \frac{1}{10} \Big/ \frac{1}{10} + \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) = \frac{1}{5}$

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