SMK ST ANTHONY SARAWAK (2006)
MATHEMATICS T (PAPER 1)

1. By using the laws of algebra of sets, prove that $(A \cup B) - (A' \cap B) = A$. [4]

2. Given that $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$, find $\mathbf{A}^3$. Hence, determine $\mathbf{A}^{15}$. [5]

3. Given that $y = \sqrt{\cos x}$. Show that $4y^3 \frac{d^2y}{dx^2} + y^4 + 1 = 0$. [6]

4. Solve the inequality $\frac{x+1}{|x-1|} < 4$. [7]

5. Express $\frac{2}{(2x-1)(2x+1)}$ in partial fractions.

   Hence, find the sum of the first n terms of the series $\frac{1}{1(3)} + \frac{1}{3(5)} + \frac{1}{5(7)} + \ldots\ldots$ [7]

6. (a) Find the distance between the two lines, $3x + 4y - 20 = 0$ and $3x + 4y - 25 = 0$. [3]

   (b) A circle passes through $A(-6, 6)$, $B(1, -1)$ and $C(-5, 7)$. Show that ABC is a right-angled triangle and hence find the equation of the circle. [5]

7. (a) The functions f and g are defined as $f : x \to e^{-x}, x > 0$ and $g : x \to \frac{1}{1-x}, x < 1$.

   Find gf and state the range of gf. Then define $(gf)^{-1}$ in a similar form. [5]

   (b) The function h is defined as $h(x) = \begin{cases} x^2 - 3x & , x \geq 3 \\ x - 3 & , x < 3 \end{cases}$.

   Determine whether h is continuous at $x = 3$. [3]

8. (a) If $(3 + \sqrt{3})$ and its conjugate are the roots of the equation $2x^2 + 3px + 3q = 0$, where p and q are constants, find the value of pq. [5]

   (b) Given that $f(x) = x^4 + ax^3 + bx^2 + 12x - 16$, where a and b are constants. If $(x - 4)$ is a factors of and $f(x)$ and $(x - 3)$ is a factor of $f'(x)$, find the values of a and b. [4]

9. (a) Using the substitution $u = 2x - 1$, evaluate $\int_1^2 \frac{x-1}{(2x-1)^2}\,dx$, giving your answer correct to 3 significant figures. [4]

   (b) The region R is bounded by the curve $y = \sqrt[4]{x^3 + 1}$, the lines $x = 1$ and $x = 3$, and the x–axis. Express as an integral the volume V of the solid formed when R is rotated completely about x–axis.

   Use the trapezium rule with 4 intervals to estimate V, giving your answer correct to 2 decimal places. [5]

10. Given that $\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 4 & 15 & 10 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 25 & -5 & -1 \\ -2 & -6 & 2 \\ -7 & 11 & -1 \end{pmatrix}$. Find $\mathbf{PQ}$ and deduce $\mathbf{P}^{-1}$.

    A school has 82 Sixth Form students in classes A, B and C. There are x students in class A, y students in class B and z students in class C. $\frac{1}{6}$ of class A, $\frac{1}{3}$ of class B and $\frac{1}{2}$ of class C students study physics. $\frac{1}{10}$ of Class A, $\frac{3}{8}$ of class B and $\frac{1}{4}$ of class C students study Biology. If there are 27 students study Physics and 19 students study Biology, obtain a system linear equations based on the information given.

    Rewrite the equations in matrix form, and solve the equations to obtain the number of students in each class. [10]

11. (a) A geometric series has first term 1 and the common ratio r is positive. The sum of the first 5 terms is twice the sum of the terms from the 6th to the 15th inclusive. Prove that $r^5 = \frac{1}{2}(\sqrt{3} - 1)$. [5]

    (b) Expand $\sqrt{\frac{1-x}{1+x}}$ as a series in ascending powers of x, up to and including the term x^2, and state the range of values of x for which the expansion is valid. Hence, by substituting $x = \frac{1}{8}$, find an approximation for $\sqrt{7}$. [6]

12. The equation of a curve is given by $y = \frac{(x+1)^2}{x^2 + x - 2}$.

    Obtain the equations of the asymptotes and determine the coordinates and nature of the stationary points on the curve.

    Sketch the curve.

    Hence determine the number of real roots of the equations

    $k(x - 1)(x + 2)^2 = (x + 1)^2$, where $k > 0$. [15]

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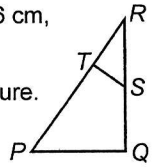
MATHEMATICS T (PAPER 1)

1. Prove that $\cos 3\theta \sin 2\theta - \cos 4\theta \sin \theta = \cos 2\theta \sin \theta$. [3]

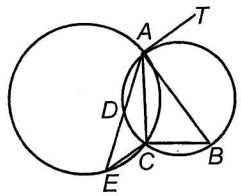
2. In the figure, $PR = 10$ cm, $RS = 3$ cm, $PQ = 6$ cm, $QS = 5$ cm and $\angle STR = \angle PQR$.



   (a) Determine two similar triangles in the figure. Give reasons. [2]

   (b) Find the lengths of TS and PT. [3]

3. In the figure, AB is a tangent to the circle ACE at A while AT is a tangent to the circle $ABCD$ at A. Prove that $\angle BAT = \angle DCE$. [6]



4. Express $\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]
   Hence, in the range $0^\circ \le \theta \le 360^\circ$, find

   (a) the values of θ which satisfy the equation $2\sin\theta + 2\sqrt{3}\cos\theta - 3 = 0$. [3]

   (b) the largest value of $\dfrac{1}{3 - \left|\sin\theta + \sqrt{3}\cos\theta\right|}$.

   Find also the values of θ where this largest value occurs. [3]

5. At 0800 on a pleasant day, an illegal fishing vessel is spotted at a point F with position vector $(17\underline{i} - 13\underline{j})$ km relative to a harbour. The fishing vessel is travelling with velocity $(3\underline{i} + 10\underline{j})$ km h^{-1}. At that instant, a coastguard vessel is at a point C with position vector $(7\underline{i} + 17\underline{j})$ km relative to the harbour and travelling with a velocity $(7\underline{i} - 2\underline{j})$ km h^{-1}. Find

   (a) the velocity of the coastguard relative to the fishing vessel in terms of $\underline{i}$ and $\underline{j}$. [2]

   (b) the vector $\overrightarrow{CF}$. [2]

   Hence, show that if the vessels continue with these velocities, the coastguard will intercept the fishing vessel. [3]

   Find also the time of interception of the two vessel. [2]

6. Newton's Law of cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and it's surrounding. A cake is taken from an oven when its temperature has reached 150°C and is placed on a table in a room where the temperature is 25°C.

   (a) If y is the temperature of the cake after t minutes, show that $\dfrac{dy}{dt} = k(y - 25)$. [1]

   (b) Find y in terms of t. [5]

   (c) If the temperature of the cake is 100°C after 10 minutes, what is its temperature after 20 minutes? [3]

   (d) How long will it takes for the cake to cool to 40°C? [2]

7. The figure shows 10 chairs arranged in four rows.
   Find the number of ways 8 guests can take their seat if

1st row	O O O O
2nd row	O O O
3rd row	O O
4th row	O

   (a) they can have free sitting. [1]

   (b) Ali wants to sit on the first row. [1]

   (c) four ladies want to sit on the fourth row. [1]

   (d) the twins want to sit on the same row. [3]

8. Four fair coins are tossed simultaneously. Find the probability of obtaining four "tails". [2]
   This process is repeated 100 times. By using a suitable approximation, find the probability of getting at least two times four "tails". [6]

9. The marks in a certain examination were normally distributed with mean μ and standard deviation σ. 5% of the candidates had more than 85 marks and 10% had less than 25 marks. Find the values of μ and σ. [6]

10. The stemplot shows the teaching experiences, in months, of 17 teachers in a school.

0	5 6 9
1	2 3 7 4 0 5
2	5 2 7
3	8 1
4	3
5	4
6	7

key : 3 | 8 means

   (a) Find the median, first quartile, and third quartile. [5]

   (b) Draw a box plot to represent the data. Comment on this distribution. [3]

11. A cabinet contains 20 shirts described in the table. Two of these shirts, which represent a random selection from the 20 shirts, have fallen off the cabinet and were lost. The events A, B, C are defined as follows.

Colour \ Size	S	M	L
Blue	1	3	6
Cream	4	2	4

   A : No medium size shirt was lost.

   B : The lost shirts include a small size cream shirt.

   C : At least one of the cream shirts was lost.

   (a) Find $P(A)$, $P(B)$, $P(A \cap B)$ and show that $P(A \cup B) = \frac{25}{38}$. [6]

   (b) Find $P(C)$ and show that $P(C \mid A) = \frac{4}{5}$. [5]

12. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} a(x^2 - x + b) & ,0 \le x \le 2 \\ 0 & ,\text{otherwise} \end{cases}$$

, where a and b are constants.

   Given that $E(X) = \frac{4}{5}$, show that $a = -\frac{3}{10}$ and find the value of b. [7]

   Show that $\text{Var}(X) = \frac{6}{25}$. [4]

   Find the cumulative distribution function of X.

   Hence, find the value of $P(1 \le X \le 2)$. [6]

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No.: ..... MT1 Date: .....

1. $(A\cup B)-(A'\cap B)=(A\cup B)\cap(A'\cap B)'$

$=(A\cup B)\cap(A\cup B')=A\cup[B\cap B']$

$=A\cup\emptyset=A$ ✱

2. $A^2=\begin{pmatrix}-1&1&-1\\2&0&1\\4&-2&-3\end{pmatrix}$, $A^3=\begin{pmatrix}1&-1&1\\2&0&1\\0&2&-1\end{pmatrix}$ ✱

$A^{15}=(A^3)^5=A^5=(A^3)A^2=AA^2$

$=A=\begin{pmatrix}1&-1&1\\2&0&1\\0&2&-1\end{pmatrix}$ ✱

3. $y=\sqrt{\cos x}\Rightarrow y^2=\cos x$

$\therefore 2yy'=-\sin x\Rightarrow 4y^2(y')^2=\sin^2 x$

$\therefore 4(y')^2=\frac{1}{y^2}(1-\cos^2 x)=\frac{1}{y^2}-y^2$

$\therefore 8y'y''=\left(-\frac{2}{y^3}-2y\right)y'$

$\therefore 4y^3\left(\frac{d^2y}{dx^2}\right)+y^4+1=0$ ✱

4. $\frac{x+1}{|x-1|}<4\Rightarrow x+1<4|x-1|$

$\therefore 4(x-1)<-(x+1)$ or $x+1<4(x-1)$

$\{x: x<\frac{3}{5}$ or $\frac{5}{3}<x\}$ ✱

5. $f(x)=\frac{2}{(2x-1)(2x+1)}\equiv\frac{A}{2x-1}+\frac{B}{2x+1}$

$\therefore 2\equiv A(2x+1)+B(2x-1)$

$x=\frac{1}{2}$: $2=2A\Rightarrow A=1$

$[x]$: $2A+2B=0\Rightarrow B=-1$

$f(x)=\frac{1}{2x-1}-\frac{1}{2x+1}$ ✱

$\frac{1}{1(3)}+\frac{1}{3(5)}+\frac{1}{5(7)}+\dots+T_n=\frac{1}{2}\sum_{x=1}^{n}f(x)$

$=\frac{1}{2}\left[\left(1+\frac{1}{3}+\frac{1}{5}+\dots+\frac{1}{2n-1}\right)-\left(\frac{1}{3}+\frac{1}{5}+\dots\right.\right.$

$\left.\left.+\frac{1}{2n-1}+\frac{1}{2n+1}\right)\right]=\frac{1}{2}\left[1-\frac{1}{2n+1}\right]=\frac{n}{2(2n+1)}$ ✱

6.(a) distance $=\frac{|1-25|}{5}-\frac{|1-20|}{5}=1$ ✱

(b) $m_{AB}\cdot m_{AC}=\left(\frac{7}{-7}\right)\left(\frac{-1}{-1}\right)=-1$

$\therefore AB\perp AC\Rightarrow\Delta ABC$ is a right $\angle\Delta$ ✱

$\therefore$ Circle with BC as diameter

Eq: $\left(\frac{y+1}{x-1}\right)\left(\frac{y-7}{x+5}\right)=-1$

$\therefore x^2+y^2-6y+4x-12=0$ ✱

7(a) $gf(x)=g(e^{-x})=\frac{1}{1-e^{-x}}$

$\therefore gf: x\to\frac{1}{1-e^{-x}}$, $x>0$ ✱

$R_{gf}=\{y: y>1\}$ ✱

$(gf)^{-1}(x)=u$: $x=gf(u)=\frac{1}{1-e^{-u}}$

$\therefore 1-e^{-u}=\frac{1}{x}\Rightarrow u=-\ln\left(1-\frac{1}{x}\right)$

$\therefore (fg)^{-1}: x\to-\ln\left(1-\frac{1}{x}\right)$, $x>1$ ✱

(b) $h(3)=3^2-9=0$, $\lim_{x\to3^-}h(x)=3-3=0$

$\lim_{x\to3^+}h(x)=3^2-9=0$

$\therefore \lim_{x\to3^-}h(x)=h(3)=\lim_{x\to3^+}h(x)$

$\therefore h$ is continuous at $x=3$. ✱

8.(a) $-\frac{3p}{2}=(3+\sqrt{3})+(3-\sqrt{3})\Rightarrow p=-4$

$\frac{3q}{2}=(3+\sqrt{3})(3-\sqrt{3})\Rightarrow q=4$

$\therefore pq=-16$ ✱

(b) $f(x)=x^4+ax^3+bx^2+12x-16$

$f'(x)=4x^3+3ax^2+2bx+12$

$f(4)=0$: $256+64a+16b+48-16=0$

$\therefore 8a+2b=-36$ — ①

$f'(3)=0$: $108+27a+6b+12=0$

$\therefore 9a+2b=-40$ — ②

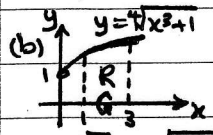
② − ①: $a=-4$ ✱ $\therefore b=-2$ ✱

9.(a) $u=2x-1$: $\frac{du}{dx}=2$

$\int_1^2\frac{x-1}{(2x-1)^2}dx=\int_1^3\frac{u-1}{2}/u^2\left(\frac{du}{2}\right)=\frac{1}{4}\int_1^3\left[\frac{1}{u}-\frac{1}{u^2}\right]du$

$=\frac{1}{4}\left[\ln u+\frac{1}{u}\right]_1^3=\frac{1}{4}\left[\ln 3+\frac{1}{3}-1\right]=0.108$ ✱

(b)



$V_x=\pi\int_1^3\sqrt{x^3+1}\,dx$ ✱

$\approx\pi\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left[\sqrt{2}+2\left(\sqrt{4.375}\right.\right.$

$\left.\left.+\sqrt{9}+\sqrt{16.625}\right)+\sqrt{28}\right]=19.67$ ✱

10. $PQ=\begin{pmatrix}16&0&0\\0&16&0\\0&0&16\end{pmatrix}$ ✱

$\therefore P^{-1}=\frac{1}{16}Q=\frac{1}{16}\begin{pmatrix}25&-5&-1\\-2&-6&2\\-7&11&-1\end{pmatrix}$ ✱

$\left.\begin{aligned}x+y+z&=82\\\frac{1}{2}x+\frac{1}{2}y+\frac{1}{2}z&=27\\\frac{1}{16}x+\frac{3}{8}y+\frac{1}{4}z&=19\end{aligned}\right\}$ $\begin{aligned}x+y+z&=82\\x+2y+3z&=162\\4x+15y+10z&=760\end{aligned}$

$\therefore P\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}82\\162\\760\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{16}\begin{pmatrix}25&-5&-1\\-2&-6&2\\-7&11&-1\end{pmatrix}\begin{pmatrix}82\\162\\760\end{pmatrix}$

$\therefore\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}30\\54\\28\end{pmatrix}\Rightarrow x=30,\ y=24,\ z=28$ ✱

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11. (a) $S_5 = 2(S_{15} - S_5) : 3S_5 = 2S_{15}$

$3\left(\frac{a(r^5-1)}{r-1}\right) = 2\left[\frac{a(r^{15}-1)}{r-1}\right]$, [Let $r^5 = u$]

$\therefore\ 2u^3 - 3u + 1 = 0 \Rightarrow (u-1)(2u^2+2u-1) = 0$

[$\because r \neq 1 \Rightarrow u \neq 1$] : $2u^2 + 2u - 1 = 0$

[$\because r > 0 \Rightarrow r^5 > 0$] : $r^5 = \frac{1}{2}(\sqrt{3}-1)$ *

(b) $f(x) = \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} = \frac{1-x}{\sqrt{1-x^2}}$

$= (1-x)\left[1 + (-\frac{1}{2})(x^2) + \dots\right] = 1 - x + \frac{1}{2}x^2 + \dots$ *

For Exp. Valid: $|-x^2| < 1 \Rightarrow |x| < 1$

$\{x : -1 < x < 1\}$ *

$f(\frac{1}{8}) = \sqrt{\frac{7}{8}/\frac{9}{8}} = \frac{1}{3}\sqrt{7}$

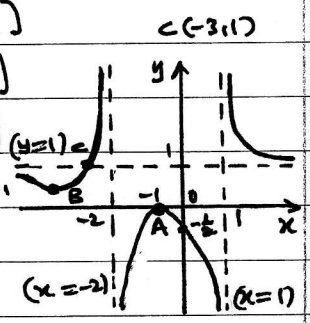
$\therefore \sqrt{7} = 3\left[1 - \frac{1}{8} + \frac{1}{2}(\frac{1}{8})^2 + \dots\right] = 2.648$ (3dp) *

12. $y = \frac{(x+1)^2}{x^2+x-2} = 1 + \frac{x+3}{(x+2)(x-1)}$

$\therefore y = 1 + \frac{1}{3}\left[\frac{4}{x-1} - \frac{1}{x+2}\right]$

$y' = \frac{1}{3}\left[\frac{-4}{(x-1)^2} + \frac{1}{(x+2)^2}\right]$

$y'' = \frac{1}{3}\left[\frac{8}{(x-1)^3} - \frac{2}{(x+2)^3}\right]$



asymptotes: $y = 1, x = -2, x = 1$ *

$y' = 0$: $\frac{x-1}{x+2} = \pm 2$

$\therefore x = -1$ or -5

$\therefore$ Stat point: $A(-1, 0)$, $B(-5, \frac{8}{9})$

$x = -1$: $y'' = -1 (<0) \Rightarrow A(-1,0)$ max *

$x = -5$: $y'' = \frac{5}{81} (>0) \Rightarrow B$ is min. *

$k(x-1)(x+2)^2 = (x+1)^2$ — ①

$\therefore k(x+2) = \frac{(x+1)^2}{x^2+x-2}$

$\therefore y = k(x+2)$ cuts the graph of $y = \frac{(x+1)^2}{x^2+x-2}$ at only 1 point for $k > 0$.

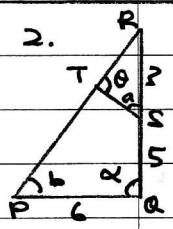
$\therefore$ ① has only 1 real root. *

1. $\cos 3\theta \sin 2\theta - \cos 4\theta \sin\theta$

$= \frac{1}{2}[\sin 5\theta - \sin\theta] - \frac{1}{2}[\sin 5\theta - \sin 3\theta]$

$= \frac{1}{2}[\sin 3\theta - \sin\theta] = \cos 2\theta \sin\theta$ *

2.



(a) $\alpha = \theta$ (given)

$\Rightarrow$ PQST is a cyclic quadrilateral (ext. ∡ = opposite vertex ∡)

$\therefore a = b$ (PQST is cyclic)

∡SRT = ∡PRQ (common ∡)

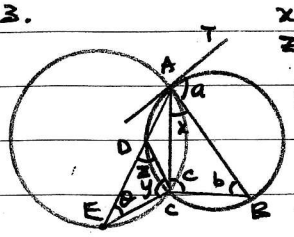
$\therefore \triangle SRT \sim \triangle PRQ$ *

(b) $\frac{TS}{QP} = \frac{SR}{PR} \Rightarrow TS = 6 \times \frac{3}{10} = 1.8$ cm *

$\frac{TR}{QR} = \frac{SR}{PR} \Rightarrow TR = 8 \times \frac{3}{10} = 2.4$

$\therefore PT = PR - TR = 10 - 2.4 = 7.6$ cm *

3.



$x = \theta$ [∡ of alternate segment]

$z = b$ [Ext ∡ of cyclic ABCD]

$x + y + z = 180°$, $x + c + b = 180°$ } ≡ ∡ in △

$\therefore y = c$

$a = c$ [∡ of alternate segment]

$\therefore y = a \Rightarrow$ ∡BAT = ∡DCE *

4. $f(\theta) = \sin\theta + \sqrt{3}\cos\theta = R\sin(\theta + \alpha)$

$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$\therefore$ $R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ } $\tan\alpha = \sqrt{3}$, $\therefore \alpha = 60°$ | $R(\frac{1}{2}) = 1$, $\therefore R = 2$

$\therefore f(\theta) = 2\sin(\theta + 60°)$ *

(a) $2\sin\theta + 2\sqrt{3}\cos\theta - 3 = 0$

$\therefore \sin(\theta + 60°) = \frac{3}{4}$

$\therefore \theta + 60° = 131.4°$ or $408.6°$

$\therefore \theta = 71.4°$ or $348.6°$ *

(b) $g(\theta) = \frac{1}{3 - |2\sin(\theta + 60°)|}$:

$g(\theta)_{max} = \frac{1}{3-2} = 1$ * when

$\sin(\theta + 60°) = \pm 1 \Rightarrow \theta + 60° = 90°, 270°$

$\therefore \theta = 30°$ or $210°$ *

5 (a) $\underline{V}_{CF} = \underline{V}_C - \underline{V}_F = \begin{pmatrix}7\\-2\end{pmatrix} - \begin{pmatrix}3\\10\end{pmatrix} = \begin{pmatrix}4\\-12\end{pmatrix}$ *

(b) $\overrightarrow{CF} = \begin{pmatrix}17\\-13\end{pmatrix} - \begin{pmatrix}7\\17\end{pmatrix} = \begin{pmatrix}10\\-30\end{pmatrix}$ *

$\overrightarrow{CF} = 10\begin{pmatrix}1\\-3\end{pmatrix} = 10\left[\frac{1}{4}\underline{V}_{CF}\right] = \frac{5}{2}\underline{V}_{CF}$

$\therefore \overrightarrow{CF} \parallel \underline{V}_{CF} \Rightarrow$ vessels meet/intercept *

$\therefore$ time taken $= \frac{5}{2}$ hours

$\therefore$ time happen $= 1030$ *

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6. (a) $\frac{dy}{dt} \propto (y-25) \Rightarrow \frac{dy}{dt} = k(y-25)$ ✱

(b) $\int \frac{dy}{y-25} = \int k\,dt \Rightarrow \ln(y-25) = kt + c$

$t=0$: $\ln 125 = c$

$\therefore \ln\left[\frac{y-25}{125}\right] = kt$

$\therefore y = 25 + 125e^{kt}$ ✱

(c) $t=10$: $\ln\frac{75}{125} = 10k \Rightarrow k = \frac{1}{10}\ln\frac{3}{5}$

$t=20$: $y = 25 + 125e^{2\ln(3/5)}$

$\therefore y = 25 + 125\left(\frac{9}{25}\right) = 70°C$ ✱

(d) $y=40$: $\ln\left(\frac{15}{125}\right) = \frac{t}{10}\ln\left(\frac{3}{5}\right)$

$\therefore t = 41.5$ minutes ✱

7. (a) No. of ways $= {}^{10}P_8 = 1814400$ ✱

(b) No. of ways $= {}^{9}P_7 = 181440$ ✱

(c) No. of ways $= {}^{6}P_4 . 4! = 8640$ ✱

(d) No. of ways $= {}^{8}P_6(2!) +$

${}^{8}C_1(3!).{}^{7}P_5 + {}^{8}C_2(4!).{}^{6}P_4 = 403200$ ✱

8. $P(4\text{'tails'}) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$ ✱

X = no. of 4'tail', $X \sim B(100, \frac{1}{16})$

$X \sim N\left(\frac{25}{4}, \frac{375}{64}\right)$

$P(X \geq 2) = P(X > 1.5) = P\left(Z > \frac{1.5 - \frac{25}{4}}{\sqrt{375/64}}\right)$

$= P(Z > -1.9623) = 0.9751$ ✱

9. $P(X > 85) = 0.05$: $\frac{85-\mu}{\sigma} = z_{0.05}$

$\therefore 85 - \mu = 1.645\sigma$ — ①

$P(X < 25) = 0.1$: $\frac{25-\mu}{\sigma} = -z_{0.1}$

$\therefore 25 - \mu = -1.282\sigma$ — ②

① − ②: $60 = 2.927\sigma \Rightarrow \sigma = 20.50$ ✱

$\therefore \mu = 85 - 1.645(20.50) = 51.28$ ✱

10.

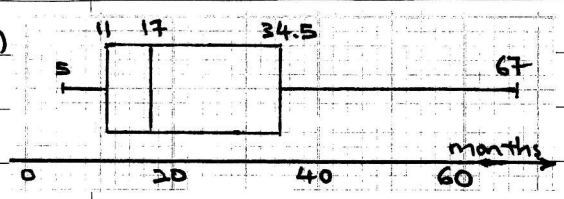
0	5 6 9
1	0 2 2 4 5 7
2	2 5 7
3	1 8
4	3
5	4
6	7

(a) $m = x_9 = 17$ ✱

$Q_1 = \frac{10+12}{2} = 11$ ✱

$Q_3 = \frac{21+38}{2} = 34.5$ ✱

(b)



Distribution: right-skewed ✱

11. (a) $P(A) = \frac{{}^{15}C_2}{{}^{20}C_2} = \frac{21}{38}$ ✱

$P(B) = \frac{{}^{4}C_1 . {}^{19}C_1}{{}^{20}C_2} = \frac{2}{5}$ ✱

$P(A \cap B) = \frac{{}^{4}C_1 . {}^{14}C_1}{{}^{20}C_2} = \frac{28}{95}$ ✱

$\therefore P(A \cup B) = \frac{21}{38} + \frac{2}{5} - \frac{28}{95} = \frac{25}{38}$ ✱

(b) $P(C) = 1 - \frac{{}^{10}C_2}{{}^{20}C_2} = \frac{29}{38}$ ✱

$P(C|A) = \left[\frac{{}^{8}C_1 . {}^{7}C_1 + {}^{8}C_2}{190}\right] \Big/ \frac{21}{38} = \frac{4}{5}$ ✱

12. $\int_0^2 a(x^2 - x + b)\,dx = 1$

$\therefore a\left[\frac{x^3}{3} - \frac{x^2}{2} + bx\right]_0^2 = 1$

$\therefore a\left[\frac{8}{3} - 2 + 2b\right] - 0 = 1$

$\therefore \frac{2a}{3}(1 + 3b) = 1$ — ①

$E(X) = \frac{4}{5}$: $\int_0^2 a(x^3 - x^2 + bx)\,dx = \frac{4}{5}$

$\therefore a\left[\frac{x^4}{4} - \frac{x^3}{3} + \frac{bx^2}{2}\right]_0^2 = \frac{4}{5}$

$\therefore a\left[4 - \frac{8}{3} + 2b\right] - 0 = \frac{4}{5}$

$\therefore \frac{2a}{3}[2 + 3b] = \frac{4}{5}$ — ②

$\frac{②}{①}$: $\frac{2+3b}{1+3b} = \frac{4}{5} \Rightarrow b = -2$ ✱ $\therefore a = -\frac{3}{10}$ ✱

$E(X^2) = \int_0^2 -\frac{3}{10}\left[x^4 - x^3 - 2x^2\right]dx$

$= -\frac{3}{10}\left[\frac{x^5}{5} - \frac{x^4}{4} - \frac{2}{3}x^3\right]_0^2$

$= -\frac{3}{10}\left[\frac{32}{5} - 4 - \frac{16}{3}\right] - 0 = \frac{22}{25}$

$\therefore Var(X) = \frac{22}{25} - \left(\frac{4}{5}\right)^2 = \frac{6}{25}$ ✱

$P(X \leq x) = \int_0^x -\frac{3}{10}(x^2 - x - 2)\,dx$

$= -\frac{3}{10}\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_0^x$

$= -\frac{3}{10}\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]$

$\therefore F(x) = \begin{cases} 0 & x < 0 \\ -\frac{3}{10}\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right] & 0 \leq x \leq 2 \\ 1 & 2 < x \end{cases}$ ✱

$P(1 \leq X \leq 2) = 1 - F(1)$

$= 1 - \left(-\frac{3}{10}\right)\left(\frac{1}{3} - \frac{1}{2} - 2\right) = \frac{7}{20}$ ✱

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