SMK ST THOMAS, KUCHING (2006)
MATHEMATICS T (PAPER 1)

1. Express $\dfrac{x^2+2x+5}{x^3-4x^2}$ in partial fractions. [5]

2. Solve the inequality $\log_2 x - 6\log_x 2 + 1 > 0$. [5]

3. Find, in terms of x and y, the gradient at the point (x, y) on the curve with equation $x^3 + 2y^3 = 3x^2y^2$. [2]

   Hence, find the point, other than the origin, at which $\dfrac{dy}{dx} = 0$. [3]

4. The function f is defined by $f(x) = \begin{cases} \dfrac{x-1}{|x-1|} & , x < 1 \\ 2 & , x = 1 \\ 4x-2 & , x > 1 \end{cases}$.

   (a) Find $\lim\limits_{x \to 1^-} f(x)$ and $\lim\limits_{x \to 1^+} f(x)$.

   Hence, determine whether the function f is continuous at $x = 1$. [3]

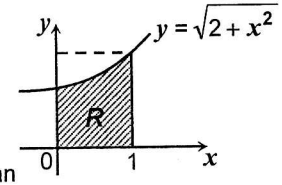
   (b) Sketch the graph of f. [2]

5. A circle touches the straight line $4y = 3x - 8$ at the point $P(4, 1)$ and passes through the point $Q(5, 3)$.

   (a) Find the centre and the radius of the circle. [6]

   (b) Find the equation of the circle. [1]

6. The diagram shows the region R bounded by the curve $y = \sqrt{2+x^2}$, the axes and the line $x = 1$.



   (a) Find the volume of the solid generated when R is rotated 360° about the x–axis. [3]

   (b) Use the trapezium rule with four intervals to obtain an approximation for the area R, giving your answer correct to 3 significant figures. [3]

7. (a) Evaluate $\displaystyle\int_3^4 \frac{3x}{\sqrt{x-2}}\,dx$. [4]

   (b) Find $\displaystyle\int \frac{(\ln\sqrt{x})^2}{x}\,dx$. [3]

8. If x is so small that terms in x^4 and higher powers of x may be neglected, prove that $\sqrt{\dfrac{1-x}{1-2x}} \approx 1 + \dfrac{1}{2}x + \dfrac{7}{8}x^2 + \dfrac{25}{16}x^3$. [6]

   By putting $x = \dfrac{1}{20}$, find $\sqrt{38}$ correct to 4 decimal places. [2]

9. (a) Given that $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 4 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ a & b \end{pmatrix}$ where $a \neq b$.

   Find the values of a and b such that $\mathbf{B}^{-1}\mathbf{AB}$ is a diagonal matrix. [8]

   (b) If $\mathbf{M} = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{pmatrix}$, show that $\mathbf{MN} = \mathbf{NM}$. [2]

   Hence, find $\mathbf{M}^{-1}$. [2]

10. Sketch on the same diagram, the graphs of $y = 3x - 1$ and $y = x^3$.
    Hence, show that the equation $3x - 1 = x^3$ has three real roots.
    Show that the equation $x^3 - 3x + 1 = 0$ has a root, α, between 0 and 1. [7]

    With $x_1 = 0.5$ as the first approximation to α, use the Newton-Raphson method to find α, correct to two decimal places. [6]

11. The polynomial $p(x) = x^4 + 5x^3 + ax^2 + bx - 30$ has the factor $(x - 2)$ and the remainder is -12 when divided by $(x - 1)$. Find the values of the constant a and b. [6]

    Show that $(x + 3)$ is a factor of $p(x)$, and find the polynomial $q(x)$ such that
    $p(x) = (x + 3)(x - 2).q(x)$. [4]

    Show that there are no real values of x for which $q(x) = 0$. [2]

    Find the set of values of x for $p(x) > 0$. [2]

12. The equation of a curve is $y = \dfrac{8x-10}{x^2-1}$.

    (a) State the equations of the asymptotes of the curve. [2]

    (b) Find the points on the x–axis and y–axis intersected by the curve. [2]

    (c) Find all the stationary points on the curve and determine whether it is a minimum point or maximum point. [7]

    (d) Sketch the curve. [2]

LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

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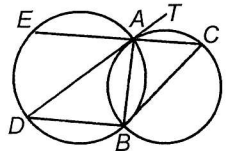
LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMK ST THOMAS, KUCHING (2006)
MATHEMATICS T (PAPER 2)

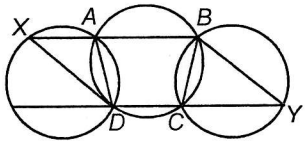
1. Solve for $0^\circ \le x \le 360^\circ$, the equation $3\cos^2 x - 2\sin x = 2$. [4]

2. Given that $\underline{p} = \lambda\underline{i} + (2\lambda - 1)\underline{j}$, $\underline{q} = (1-\lambda)\underline{i} + 3\lambda\underline{j}$, where λ is a scalar.
   (a) Find the values of λ if $\underline{p}$ and $\underline{q}$ are perpendicular. [4]
   (b) If $\lambda = 2$, $\underline{p}$ and $\underline{q}$ are the position vectors of P and Q respectively, relative to a fixed origin O,
      (i) find $\overrightarrow{PQ}$, [2]
      (ii) find $\angle QPO$, to the nearest degree. [4]

3. Given that ABC and $ABDE$ are 2 circles which intersect at A and B. DAT is a tangent to the circle ABC and CBS is a tangent to the circle $ABDE$. EAC is a straight line. Show that EC is parallel to DB. [4]



4. Given that the radii of the circles XAD, $ABCD$ and BYC are equal. $AD = BC$, XAB and DCY are straight lines. Show that
   (a) AB is parallel to CD, [2]
   (b) triangles XAD and DCB are congruent. [5]



5. A bag contains 7 red marbles and 3 blue marbles. A marble is selected at random. A different coloured marble is put back into the bag as a replacement. The second marble is then selected. Draw a tree diagram. [2]
   (a) Find the probability that the second marble is red. [2]
   (b) If both marbles are the same colour, find the probability that both are blue. [3]

6. Solve the differential equation $x^2\frac{dy}{dx} + xy - y^2 = 0$ by substituting $y = \frac{u}{x}$, given $y = 2$ when $x = 1$. Give your answer in a form expressing y in terms of x. Hence, sketch the curve. [6]

7. A random number generator in a certain game produces values which can be modelled by the discrete random variable R whose probability function is given by
   $P(R = r) = k(r!)$, for $r = 0, 1, 2, 3, 4$ where k is a constant.
   (a) Show that $k = \frac{1}{34}$. [2]
   (b) Find $E(R)$ and $Var(R)$. [4]
   (c) Given random variable S with $S = 3R + 4$ find $E(S)$ and $Var(S)$. [4]

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8. The continuous random variable X has probability density function, f given by
$$f(x) = \begin{cases} c & ,0 \le x < 2 \\ c(3-x) & ,2 < x \le 3 \\ 0 & ,\text{otherwise} \end{cases}$$
, where c is a constant.
   (a) Show that $c = \frac{2}{5}$. [3]
   (b) Sketch the graph of f. [2]
   (c) Find k such that $P(X \le k) = \frac{9}{10}$, giving your answer correct to 3 decimal places. [5]

9. The editor of a newspaper decides to change the paper to tabloid form. He claims that 25% of its reader favour the new format in preference to the old format.
   (a) Assuming that this figure is correct, calculate the probability that in a random sample of 6 readers at least 2 of them prefer the new format. [4]
   (b) Using a suitable approximation calculate the probability that in a random sample of 250 readers between 45 and 70 prefer the new format. [5]

10. Given that $f(x) = 5\cos^2 x + \sin^2 x + 3\sin x\cos x$.
    Show that $f(x)$ can be expressed as $3 + \sin\left(2x + \tan^{-1}\frac{4}{3}\right)$.
    Hence, find the largest and least values of $f(x)$. [8]

11. The depreciation in the value of a second-hand car is modelled by assuming that the rate of change in the value, x in RM at any time t, in months, after the car is brought is proportional to x. The car was brought for RM 50 000 when $t = 0$.
    (a) Show that $x = 50\,000e^{-kt}$, where k is a constant. [4]
    (b) After 24 months the car values of the car is RM30 000. Find
       (i) the value of the car to the nearest RM after 36 months, [4]
       (ii) how long to the nearest month before the car is worth just RM10 000. [3]

12. The marks obtained by 51 students in a class taking Mathematics examination paper are given below.

47	45	54	28	60	77	75	35	82	42	72	44	73	46	32	57	52
56	71	74	34	48	61	43	66	38	23	41	44	84	27	61	80	63
43	21	58	29	51	30	54	74	45	35	48	36	24	37	45	65	37

   (a) Construct an ordered stem and leaf plot to represent these data and determined the values of the quartiles and the mean value. [6]
   (b) Construct a box and whiskers plot and comment on the skewness of the distribution. Determine if there are any outliers. [5]
   (c) State the mode of the distribution and the semi-interquartile range. [3]

1. $f(x) = \dfrac{x^2+2x+5}{x^3-4x^2} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-4}$

$\equiv \dfrac{Ax(x-4)+B(x-4)+Cx^2}{x^2-4x^2}$

$\therefore x^2+2x+5 \equiv Ax(x-4)+B(x-4)+Cx^2$

$x=0$: $5 = B(-4) \Rightarrow B = -\frac{5}{4}$

$x=4$: $16+8+5 = C(16) \Rightarrow C = \frac{29}{16}$

$[x^2]$: $1 = A+C \Rightarrow A = -\frac{13}{16}$

$\therefore f(x) = \frac{1}{16}\left[\frac{29}{x-4} - \frac{13}{x} - \frac{20}{x^2}\right]$ ✱

2. Let $\log_2 x = u$: $\log_2 x - \log_x 2 + 1 > 0$

$\therefore u - \frac{2}{u} + 1 > 0 \Rightarrow \frac{u^2+u-2}{u} > 0$

$\therefore u(u-1)(u+2) > 0$

$\therefore -2 < u < 0$ or $1 < u$

$\therefore -2 < \log_2 x < 0$ or $1 < \log_2 x$

$\therefore \{x : \frac{1}{4} < x < 1$ or $2 < x\}$ ✱

3. $x^3 + 2y^3 = 3x^2y^2$ — ①

$\therefore 3x^2 + 6y^2y' = 3x^2(2yy') + y^2(6x)$

$\therefore y' = \dfrac{2xy^2 - x^2}{2y^3 - 2x^2y}$ ✱

$y'=0$: $2xy^2 - x^2 = 0 \Rightarrow x(2y^2 - x) = 0$

$[\because x \neq 0]$: $x = 2y^2$

①: $2y^6 + 2y^3 = 3y^2(4y^4)$

$\therefore y^3 = \frac{1}{2}$ $[\because y \neq 0]$

$\therefore$ Point $= (\sqrt[3]{2}, \frac{1}{\sqrt[3]{2}})$ ✱

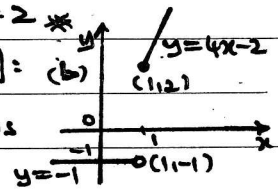
4.(a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left[\frac{x-1}{1-x}\right] = -1$ ✱

$\lim_{x\to 1^+} f(x) = 4(1) - 2 = 2$ ✱

$[\because \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)]$:

$\therefore f$ is not continuous at $x = 1$ ✱

(b)



5(a) PC: $4x + 3y = 4(4) + 3(1) = 19$ — ①

$\perp$ bisector (PQ): $(x-4)^2 + (y-1)^2 = (x-5)^2 + (y-3)^2$

$\therefore 2x + 4y = 17$ — ②

②×2 − ①: $5y = 15 \Rightarrow y = 3$

$\therefore$ centre: $C(\frac{5}{2}, 3)$ ✱

$r = CP = \left[(-\frac{3}{2})^2 + 2^2\right]^{1/2} = \frac{5}{2}$ ✱

(b) Eq: $(x - \frac{5}{2})^2 + (y-3)^2 = (\frac{5}{2})^2$

$\therefore x^2 + y^2 - 5x - 6y + 9 = 0$ ✱

6.(a) $V_x = \pi\int_0^1 (2+x^2)\,dx = \pi\left[2x + \frac{x^3}{3}\right]_0^1$

$= \pi\left[2 + \frac{1}{3}\right] - 0 = \frac{7\pi}{3}$ ✱

(b) Area $\approx \frac{1}{2}(\frac{1}{4})\left[\sqrt{2} + 2\left(\sqrt{\frac{33}{16}} + \frac{3}{2} + \frac{5}{4}\right) + \sqrt{3}\right]$

$= 1.44$ ✱

7.(a) $\int_3^4 \frac{3x}{\sqrt{x-2}}\,dx = 3\int_3^4 \left[\frac{x-2+2}{\sqrt{x-2}}\right]dx$

$= 3\int_3^4 \left(\sqrt{x-2} + \frac{2}{\sqrt{x-2}}\right]dx$

$= \left[2(x-2)^{3/2} + 12(x-2)^{1/2}\right]_3^4$

$= \left[4\sqrt{2} + 12\sqrt{2}\right] - \left[2+12\right] = 16\sqrt{2} - 14$ ✱

(b) Let $\ln x = u$: $\frac{du}{dx} = \frac{1}{x}$

$\int \frac{(\ln\sqrt{x})^2}{x}\,dx = \int \frac{(\ln x)^2}{4x}\,dx$

$= \int \frac{u^2}{4x}(x\,du) = \int \frac{1}{4}u^2\,du = \frac{1}{12}u^3 + c$

$= \frac{1}{12}(\ln x)^3 + c$ ✱

8. $\sqrt{\frac{1-x}{1-2x}} = \left[1 + \frac{1}{2}(-x) + \frac{1}{2}(\frac{1}{2})(-\frac{1}{2})(-x)^2\right.$

$\left. + \frac{1}{6}(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-x)^3 + \dots\right]\left[1 + (-\frac{1}{2})(-2x)\right.$

$\left. + \frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-2x)^2 + \frac{1}{6}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-2x)^3 + \dots\right]$

$= \left[1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots\right]\left[1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \dots\right]$

$\approx 1 + \frac{1}{2}x + \frac{7}{8}x^2 + \frac{25}{16}x^3$ ✱ $[= f(x)]$

$f(\frac{1}{20}) = \sqrt{\frac{19}{20} / \frac{9}{10}} = \frac{1}{6}\sqrt{38}$

$\therefore \sqrt{38} = 6\left[1 + \frac{1}{2}(\frac{1}{20}) + \frac{7}{8}(\frac{1}{20})^2 + \frac{25}{16}(\frac{1}{20})^3 + \dots\right]$

$= 6.1643$ (3dp) ✱

9.(a) Let $D = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$: $B^{-1}AB = D$

$\therefore AB = BD$: $\begin{pmatrix} -1 & 2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ a & b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ a & b \end{pmatrix}\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$

$\therefore \begin{pmatrix} -1+2a & -1+2b \\ 4+a & 4+b \end{pmatrix} = \begin{pmatrix} x & y \\ ax & by \end{pmatrix}$

$-1+2a = x$ and $-1+2b = y$

$4+a = ax$ $\quad$ $4+b = by$

$\therefore 4+a = a(-1+2a)$ $\quad$ $4+b = b(-1+2b)$

$\therefore a^2 - a - 2 = 0$ $\quad$ By the same method:

$\therefore (a+1)(a-2) = 0$ $\quad$ $b = -1$ or 2

$\therefore a = -1$ or 2

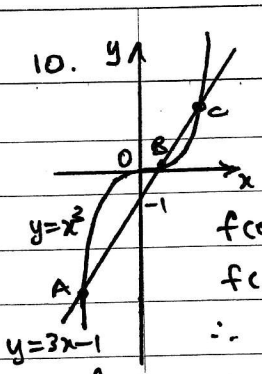
$[\because B^{-1}$ exist $\Rightarrow a \neq b]$: $a = -1, b = 2$ or $a = 2, b = -1$ ✱

(b) $MN = \begin{pmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{pmatrix}$, $NM = \begin{pmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{pmatrix}$

$\therefore MN = NM = 18I$ ✱

$\therefore M^{-1} = \frac{1}{18}N = \frac{1}{18}\begin{pmatrix} \frac{1}{5} & \frac{-5}{7} & \frac{7}{5} \\ \frac{-1}{5} & \frac{1}{7} & \frac{-1}{5} \end{pmatrix}$ ✱

LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

10. 

$3x-1=x^3$ — ①

$y=3x-1$ cuts $y=x^3$ at 3 points

$\Rightarrow$ ① has 3 real roots *

Let $f(x)=x^3-3x+1$:

$f(0)\cdot f(1)=(1)(-3)=-3<0$

$f(1)\cdot f(2)=(-3)(3)=-9<0$

$\therefore x_B\in(0,1),\ x_C\in(1,2)$

$\therefore f(x)=0$ has a root, $\alpha\in(0,1)$ *

By NRM: $x'=x-\dfrac{x^3-3x+1}{3x^2-3}$

$\therefore x'=\dfrac{2x^3-1}{3x^2-3}$

$x_1=0.5$: $x_2=\dfrac{2(0.5)^3-1}{3(0.5)^2-3}$

$=0.3333\simeq 0.33$

$x_3=0.3472\simeq 0.35$

$x_4=0.3473\simeq 0.35$

$\therefore \alpha=0.35$ (2 dp) *

11. $P(2)=0$: $16+40+4a+2b-30=0$

$\therefore 2a+b=-12$ — ①

$P(-1)=-12$: $1-5+a-b-30=-12$

$\therefore a-b=22$ — ②

① + ②: $3a=9\Rightarrow a=3\quad \therefore b=-19$ *

$\therefore P(-3)=81-135+3(9)-19(-3)-30=0$

$\therefore (x+3)$ is a factor of $P(x)$ *

$\therefore P(x)=x^4+5x^3+3x^2-19x-30$

$\equiv (x+3)(x-2)q(x)$

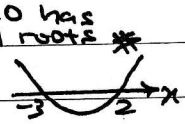
$\equiv (x^2+x-6)(x^2+4x+5)$

$\therefore q(x)=x^2+4x+5$ *

$\therefore q(x)=(x+2)^2+1$

$\therefore q(x)\geq 1,\ \forall x\in\mathbb{R}\Rightarrow q(x)=0$ has no real roots *

$P(x)\geq 0$: $(x+3)(x-2)\geq 0$



$\therefore \{x: x\leq -3 \text{ or } 2\leq x\}$ *

12. $y=\dfrac{8x-10}{x^2-1}=\dfrac{9}{x+1}-\dfrac{1}{x-1}$

(a) asymptotes: $y=0,\ x=1,\ x=-1$ *

(b) $(0,10)$, $(\frac{5}{4},0)$ *

(c) $y'=\dfrac{-9}{(x+1)^2}+\dfrac{1}{(x-1)^2}$, $y''=\dfrac{18}{(x+1)^3}-\dfrac{2}{(x-1)^3}$

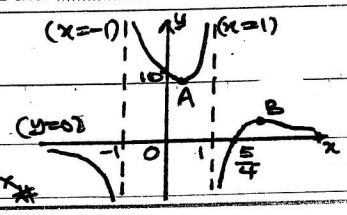
For $y'=0$: $\dfrac{x+1}{x-1}=\pm 3\Rightarrow x=\frac{1}{2}$ or 2

$\therefore$ Stat. points $=A(\frac{1}{2},8),\ B(2,2)$ *

$x=\frac{1}{2}$: $y''=\frac{64}{3}\ (>0)$

$x=2$: $y''=-\frac{4}{3}\ (<0)$

$\therefore A(\frac{1}{2},8)$ min, $B(2,2)$ max *



1. $3\cos^2x-2\sin x=2\Rightarrow 3(1-s^2)-2s=2$

$\therefore 3s^2+2s-1=0\Rightarrow (3s-1)(s+1)=0$

$\therefore \sin x=\frac{1}{3}$ or $-1\Rightarrow x=19.5^\circ, 160.5^\circ, 270^\circ$ *

2(a) $\underline{p}\perp\underline{q}$: $\begin{pmatrix}\lambda\\2\lambda-1\end{pmatrix}\cdot\begin{pmatrix}1-\lambda\\3\lambda\end{pmatrix}=0$

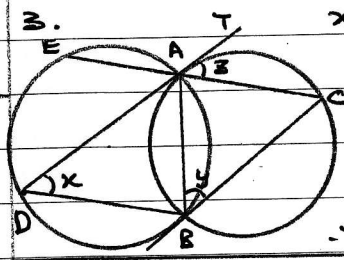
$\therefore \lambda-\lambda^2+6\lambda^2-3\lambda=0\Rightarrow 5\lambda^2-2\lambda=0$

$\therefore \lambda=0$ or $\frac{2}{5}$ *

(b) $\overrightarrow{PQ}=\underline{q}-\underline{p}=\begin{pmatrix}-1\\6\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\3\end{pmatrix}$ *

Let $\angle QPO=\theta$: $\begin{pmatrix}-3\\3\end{pmatrix}\cdot\begin{pmatrix}-2\\-3\end{pmatrix}=3\sqrt{2}\sqrt{13}\cos\theta$

$\therefore \cos\theta=\dfrac{6-9}{3\sqrt{26}}\Rightarrow\theta=101.3^\circ$ *

3. 

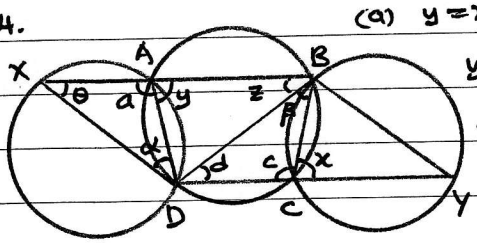
$x=y$ [$\angle$ of alternate segment]

$y=z$ [$\angle$ of alternate segment]

$\therefore x=z$

$\Rightarrow$ a pair of equal corresponding $\angle$.

$\therefore EC\parallel DB$ *

4. 

(a) $y=x$ [Ext. $\angle$ of cyclic ABCD]

$y=z+\beta$ [given $AD=BC$]

$\therefore x=z+\beta$

$\Rightarrow$ a pair of equal alternate angles

$\therefore AB\parallel DC$ *

(b) $a=c$ [Ext $\angle$ of cyclic ABCD]

$\therefore XD=BD$ [chords in same radii circles]

$\therefore \theta=z$ [ΔDBX is isosceles]

$d=z$ [alternate $\angle$, $AB\parallel DC$]

$\alpha=y-\theta$ [Ext. $\angle$ of Δ]

$\beta=x-d$ [Ext $\angle$ of Δ]

$\therefore \beta=y-\theta=\alpha$

$\therefore \Delta XAD\equiv\Delta DCB$ [by SAS] *

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5. $0.7\ R_1 \begin{cases} 0.6\ R_2 \\ 0.4\ B_2 \end{cases}$; $0.3\ B_1 \begin{cases} 0.8\ R_2 \\ 0.2\ B_2 \end{cases}$

(a) $P(R_2) = (0.7)(0.6) + (0.3)(0.8) = 0.66$ ✱

(b) $P(2B \mid \text{same colour}) = \dfrac{0.3(0.2)}{0.3(0.2)+0.7(0.6)}$

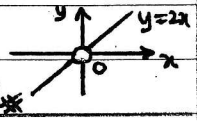
$= \frac{1}{8}$ ✱

6. $y = \frac{u}{x}$: $y' = \frac{xu' - u}{x^2} \Rightarrow x^2y' = xu' - u$

$\therefore\ xu' - u + (u) - \left(\frac{u}{x}\right)^2 = 0 \Rightarrow \int \frac{du}{u^2} = \int \frac{dx}{x^3}$

$\therefore\ -\frac{1}{u} = -\frac{1}{2x^2} + c \Rightarrow \frac{1}{xy} = \frac{1}{2x^2} + A$

$(1,2)$: $\frac{1}{2} = \frac{1}{2} + A \Rightarrow A = 0$



$\therefore\ \frac{1}{xy} = \frac{1}{2x^2} \Rightarrow y = 2x \quad (x \neq 0)$ ✱

7. (a) $\Sigma P(R=r) = 1 : k(1+1+2+6+24) = 1$

$\therefore\ k = \frac{1}{34}$ ✱

(b) $E(R) = \frac{1}{34}[0 + 1 + 2(2) + 3(3!) + 4(4!)]$

$= \frac{7}{2}$ ✱

$E(R^2) = \frac{1}{34}[0 + 1 + 2^2(2) + 3^2(3!) + 4^2(4!)] = \frac{447}{34}$

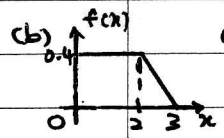
$\therefore\ Var(R) = \frac{447}{34} - \left(\frac{7}{2}\right)^2 = \frac{61}{68}$ ✱

(c) $E(S) = 3\left(\frac{7}{2}\right) + 4 = \frac{29}{2}$ ✱

$Var(S) = 3^2\left(\frac{61}{68}\right) = \frac{549}{68}$ ✱

8. (a) $c(2) + \frac{1}{2}(1)[c+0] = 1 \Rightarrow c = \frac{2}{5}$ ✱

(b)



(c) $\frac{1}{2}(3-k).\left(\frac{2}{5}\right)(3-k) = \frac{1}{10}$

$3 - k = \frac{1}{\sqrt{2}} \quad [\because k < 3]$

$\therefore\ k = 3 - \frac{1}{\sqrt{2}} = 2.293$ (3dp) ✱

9. (a) $P = 1 - [(0.75)^6 + 6(0.75)^5(0.25)]$

$= \frac{1909}{4096}$

(b) X = no. of reader prefer new format

$X \sim B(250, \frac{1}{4}) \rightarrow X \sim N\left(\frac{125}{2}, \frac{375}{8}\right)$

$\therefore\ P(45 < X < 70) = P(45.5 < X < 69.5)$

$= P\left(Z > \frac{45.5 - 62.5}{\sqrt{46.875}}\right) - P\left(Z > \frac{69.5 - 62.5}{\sqrt{46.875}}\right)$

$= 0.99349 - 0.15329 = 0.8402$ ✱

10. $f(x) = 5\cos^2 x + \sin^2 x + 3\sin x\cos x$

$= 4\left[\frac{1+\cos 2x}{2}\right] + 1 + 3\left[\frac{\sin 2x}{2}\right]$

$= \frac{3}{2}\sin 2x + 2\cos 2x + 3$

$\frac{3}{2}\sin 2x + 2\cos 2x = R\sin(2x + \alpha)$

$= R\sin 2x\cos\alpha + R\cos 2x\sin\alpha$

$\therefore \left.\begin{array}{l} R\cos\alpha = 3/2 \\ R\sin\alpha = 2 \end{array}\right\} \tan\alpha = \frac{4}{3},\ R\left(\frac{4}{5}\right) = 2$

$\therefore\ f(x) = \frac{5}{2}\sin\left[2x + \tan^{-1}\left(\frac{4}{3}\right)\right] + 3$ ✱

$\therefore\ f(x)_{max} = 3 + \frac{5}{2}(1) = \frac{11}{2}$ ✱

$f(x)_{min} = 3 + \frac{5}{2}(-1) = \frac{1}{2}$ ✱

11. $-\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = -kx \quad (k > 0)$

(a) $\int \frac{1}{x}dx = \int -k\,dt \Rightarrow \ln x = -kt + c$

$t = 0$: $\ln 50000 = c \Rightarrow \ln\left(\frac{x}{50000}\right) = -kt$

$\therefore\ x = 50000e^{-kt}$ ✱

(b) $t = 24$: $\ln\left[\frac{30000}{50000}\right] = -24k \Rightarrow k = -\frac{1}{24}\ln\left(\frac{3}{5}\right)$

(i) $t = 36$: $\ln\left(\frac{x}{50000}\right) = \frac{36}{24}\ln\left(\frac{3}{5}\right)$

$\therefore\ x = 50000\left(\frac{3}{5}\right)^{3/2} = 23238$ ✱

(ii) $x = 10000$: $\frac{t}{24}\ln\left(\frac{3}{5}\right) = \ln\left(\frac{10000}{50000}\right)$

$\therefore\ t = 75.6 \approx 76$ months ✱

12. (a)

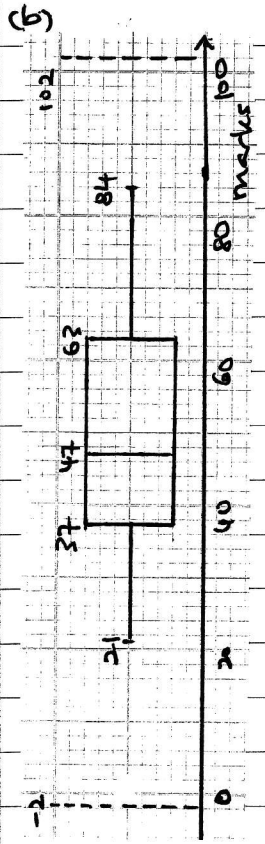
2	1 3 4 7 8 9
3	0 2 4 5 5 6 7 7 8
4	1 2 3 3 4 4 5 5 5 6 7 8 8
5	1 2 4 4 6 7 8
6	0 1 1 3 5 6
7	1 2 3 4 4 5 7
8	0 2 4

key: 2|1 means 21

$Q_1 = x_{13} = 37$, $Q_3 = x_{39} = 63$ ✱

$\bar{x} = \frac{\Sigma x}{n} = \frac{3567}{51} = 50\frac{1}{3}$ ✱

(b)



Distribution: Right-skewed (no outlier) ✱

(c) Mode = 45 ✱

$SIQR = \frac{1}{2}(63 - 37)$

$= 13$ ✱