LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK CHUNG HWA, KOTA BAHRU (2006)
MATHEMATICS T (PAPER 1)

1. Using the laws of algebra of sets, show that, for any set A, B and C

   $(A \cup B') \cap (A' \cup B) \cap (A \cup B) = A \cap B$. [4]

2. If $\mathbf{A} = \begin{pmatrix} 4 & 4 & 1 \\ -3 & -3 & a \\ b & c & d \end{pmatrix}$ and $\mathbf{A}^2 = \mathbf{I}$, where $\mathbf{I}$ is the 3 x 3 identity matrix,

   find the values of a, b, c and d. [5]

3. Expand $\dfrac{1}{\sqrt{1+y}}$ in ascending powers of y up to the term in y^4.

   Hence, by putting $y = x - 3x^2$, show that if x is small,

   $$\frac{1}{\sqrt{1+x-3x^2}} \approx 1 - \frac{1}{2}x + \frac{15}{8}x^2 - \frac{41}{16}x^3 + \frac{827}{128}x^4.$$ [7]

   By substituting $x = \frac{1}{5}$, find the approximation for $\sqrt{3}$, correct to 3 significant figures. [3]

4. If $z_1 = \dfrac{a}{1+i}$ and $z_2 = \dfrac{b}{1+2i}$, where a and b are real and if $z_1 + z_2 = 1$, find a and b. [5]

5. If the straight line $y = mx + c$ is a tangent to the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$,

   show that $c^2 = a^2m^2 + b^2$. [5]

   Find the gradients of the tangents to the curve which pass through the point $(\sqrt{a^2+b^2}, 0)$. [3]

   Hence, determine the coordinates of the vertices of the square whose sides touch the curve. [2]

6. Function f is defined by $f : x \longrightarrow \dfrac{3x+2}{x-3}$, $x \neq 3$.

   Find f^2 and deduce f^{-1}. [4]

   Hence, obtain the value of $f^{17}(4)$. [3]

7. Given that the 3rd, 5th and 8th terms of a progression are $\log ar^2$, $\log ar^4$ and $\log ar^7$ respectively. Determine whether it is an arithmetic or geometric progression. [4]

   Hence, find $\sum_{k=n+1}^{2n} U_k$, where is the U_k is the kth term of this progression. [4]

27

8. Given that $y = x + \dfrac{4}{x^2}$, show that

   (a) it lies completely above the line $y = x$,

   (b) there is only one value of x such that $\dfrac{dy}{dx} = 0$. [4]

   R is the region bounded by the curve and the line $2x + y - 7 = 0$. Find the area of **R**. [6]

   Calculate the volume of the solid formed by revolving the region **R** about the x–axis. [5]

9. Given that $y = e^{\sqrt{1+x}}$, show that $4(1+x)\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - y = 0$. [5]

10. Using a suitable substitution, show that $\displaystyle\int_0^1 \frac{e^x}{e^x+1}\,dx = \ln\left(\frac{e+1}{2}\right)$. [5]

    Hence, deduce the value of $\displaystyle\int_0^1 \frac{1}{e^x+1}\,dx$. [3]

11. A curve **C** has equation $y = \dfrac{4}{x+1} - \dfrac{1}{x-1}$.

    (a) Write down the equations of asymptotes of **C**. [2]

    (b) Find the coordinates of the turning points of **C** and determine its nature. [7]

    (c) Sketch the curve. [4]

12. Show that the equation $\ln x + x - 3 = 0$ has a root between 1 and e. [2]

    Show that the Newton-Raphson iterative formula for this root is given by

    $$x_{n+1} = \frac{x_n(4 - \ln x_n)}{1 + x_n}.$$ [5]

    Hence, find this root, correct to three significant figures. [3]

SMJK CHUNG HWA, KOTA BAHRU (2006)
MATHEMATICS T (PAPER 2)

1. Solve the equation $\sin 3\theta + \sin 5\theta = \cos\theta$ for values of θ between 0 and 2π. [5]

2. Prove the following identities,

   (a) $2\cos\theta(\sin 3\theta - \sin\theta) = \sin 4\theta$, [3]

   (b) $4(\cos 4\theta + \cos 2\theta)\sin 3\theta \sin\theta = \cos 4\theta - \cos 8\theta$. [3]

Hence, without using tables or calculators, show that

(c) $\sin 54^\circ - \sin 18^\circ = \frac{1}{2}$, [3]

(d) $\sin 54^\circ \sin 18^\circ = \frac{1}{4}$. [3]

3. (a) The variables v and t are connected by $\frac{dv}{dt} = (t-1)(v-3)$, where $v > 3$ and $t \neq 1$.

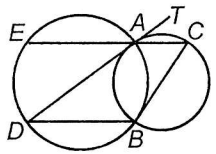
Given that $v = 4$ when $t = 0$, find v in terms t.

Hence, show that $v > 0$ for all values of t. [6]

(b) Waste discharge in a factory is channelled into a tank. Every 45 minutes, the waste, if it is below a safe level, will then be emptied into the river. Initially, a certain amount of waste will be trapped in the tank. The rate of waste discharge varies with the amount of waste in the tank itself, as well as time. The rate is $t^2 e^{-3x}$ tones per hour, where x represents the amount in the tank and t the time. At the initial stage of monitoring the waste, the amount of waste in the tank is 0.25 tones. Obtain the equation relating x to t.

Hence determine the amount of waste 0.5 hour after the initial reading. [5]

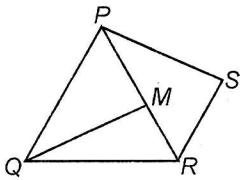
4. *ABC* and *ABDE* are two circles which intersect at *A* and *B*. *DAT* is a tangent to the circle *ABC* and *CB* is a tangent to the circle *ABDE*.



(a) Show that *EC* // *DB*.

(b) If $\angle EAT = 148^\circ$ and $\angle ACB = 56^\circ$, find $\angle BAC$ and $\angle EDA$. [7]

5. In the diagram, *PQR* is an equilateral triangle. *QP* is parallel to *RS* and *PM* = *RS*.



(a) Prove that the triangles *PQM* and *RPS* are congruent.

(b) Given also that $\angle MQR = 21^\circ$, find $\angle RPS$. [6]

6. Ship A sails at a velocity of $(30\underline{i} + 40\underline{j})$ km h^{-1} and B sails at a velocity of $(15\underline{i} + 12\underline{j})$ km h^{-1}. At a certain, the displacement vector of ship B relative to ship A is $(5\underline{i} + 12\underline{j})$ km. Calculate the magnitude and direction of the velocity of A relative to B. Find also the nearest distance between ship A and ship B in their subsequent motion. [7]

At the instant when ship A is to the East of ship B, ship A changes its direction while maintaining its speed so as to intercept B. Find the direction which must be taken by ship A and the time taken to intercept B. [8]

7. Given that where $X \sim N(20, 25)$ and $Y \sim N(\mu, \sigma^2)$ are independent random variables. If $P(X + Y > 75) = 0.06$ and $P(X - Y > 10) = 0.02$, calculate the values of μ and σ, correct to 3 significant figures. [6]

28

8. An unbiased cubical die has the number 1 on three faces, the number 2 on two faces and the number 3 on one face. The die is tossed twice and X is the total score. Find the probability distribution of X. [2]

Show that $E(X) = \frac{10}{3}$ and $Var(X) = \frac{10}{9}$. [5]

9. Based on previous records, the probability an inter-state football game ends in a draw is 0.42. The Football Association schedules 120 inter-state games this season. What is the probability at least 45 games end in draws. [5]

10. The closing prices of 25 counters traded in the Stock Exchange on a particular day are as shown in the stem plot below.

01	94					
02	40	50	78	81	84	94
03	18	20	40			
04	36	40	52	80	88	96
05	55	60				
06	00	90				
07	50					
08	15					
09	10					
10	70					
11	89					

key : 01 | 94 means RM 1.94

LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

(a) Find the median and the interquartile range. [3]

(b) Construct a frequency distribution table with class intervals :

1.90 – 3.89, 3.90 – 5.89, 5.90 – 7.89, 7.90 – 9.89, 9.90 – 11.89.

(i) Calculate the mode of the distribution. [3]

(ii) Draw a cumulative relative frequency curve and hence estimate the percentage of the closing price less than the mode. [4]

11. The cumulative distribution function of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ x - \frac{1}{4}x^2 & , 0 \le x \le a \\ 1 & , a < x \end{cases}$$, where a is a constant.

Find the value of a and the probability density function of X.

The continuous random variable Y is given by $Y = 1 - \frac{1}{2}X$.

Find $P(Y < y)$ for $0 \le y \le 1$.

Deduce the probability density function of Y and hence verify that $E(X) = E(Y)$. [11]

12. Events X and Y are such that $P(X) = 0.3$, $P(Y' | X) = 0.8$ and $P(Y | X') = 0.4$. By using a tree diagram, find $P(X \cap Y)$ and $P(X \cup Y)$. [5]

1. $(A\cup B')\cap(A'\cup B)\cap(A\cup B)$

$= (A\cup B')\cap[(A'\cap A)\cup B]$

$= (A\cup B')\cap[\emptyset\cup B] = (A\cup B')\cap B$

$= (A\cap B)\cup(B'\cap B) = (A\cap B)\cup\emptyset$

$= A\cap B$ ✱

2. $A^2 = I$: $(1,1)$: $16-12+b=1 \Rightarrow b=-3$ ✱

$(1,2)$: $16-12+c=0 \Rightarrow c=-4$ ✱

$(2,1)$: $-12+9+ab=0 \Rightarrow a=-1$ ✱

$(1,3)$: $4+4a+d=0 \Rightarrow d=0$ ✱

3. $\frac{1}{\sqrt{1+y}} = (1+y)^{-\frac{1}{2}} = 1+(-\frac{1}{2})y+\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})y^2$

$+\frac{1}{6}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})y^3+\frac{1}{24}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})y^4$

$+\dots = 1-\frac{1}{2}y+\frac{3}{8}y^2-\frac{5}{16}y^3+\frac{35}{128}y^4+\dots$ ✱

$y = x-3x^2$: $\frac{1}{\sqrt{1+x-3x^2}} = 1-\frac{1}{2}(x-3x)$

$+\frac{3}{8}(x^2-6x^3+9x^4)-\frac{5}{16}(x^3-9x^4+\dots)$

$+\frac{35}{128}(x^4+\dots)+\dots = f(x)$

$\approx 1-\frac{1}{2}x+\frac{15}{8}x^2-\frac{41}{16}x^3+\frac{827}{128}x^4$ ✱

$x=\frac{1}{5}$: $f(\frac{1}{5}) = \frac{1}{\sqrt{1+\frac{1}{5}-\frac{3}{25}}} = \frac{5\sqrt{3}}{9}$

$\therefore \sqrt{3} = \frac{9}{5}\left[1-\frac{1}{2}(\frac{1}{5})+\frac{15}{8}(\frac{1}{5})^2-\frac{41}{16}(\frac{1}{5})^3+\frac{827}{128}(\frac{1}{5})^4+\dots\right]$

$= 1.74$ (3sf) ✱

4. $z_1+z_2=1$: $\frac{a(1-i)}{2}+\frac{b(1-2i)}{5}=1$

$\therefore \frac{a}{2}+\frac{b}{5}=1$ and $-\frac{a}{2}-\frac{2b}{5}=0$

$\therefore a=-\frac{4b}{5}$

$\therefore -\frac{2b}{5}+\frac{b}{5}=1 \Rightarrow b=-5,\ a=4$ ✱

5. $y=mx+c$ — ① $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ — ②

① → ②: $b^2x^2+a^2(mx+c)^2=a^2b^2$

$\therefore (b^2+a^2m^2)x^2+2mca^2x+a^2(c^2-b^2)=0$

For tangent: $(2mca^2)^2=4(b^2+a^2m^2)a^2(c^2-b^2)$

$\therefore m^2c^2a^4 = a^2(b^2c^2-b^4+a^2m^2c^2-a^2b^2m^2)$

$\therefore a^2b^2(c^2-a^2m^2-b^2)=0$

$[\because a,b\neq 0]$: $c^2=a^2m^2+b^2$ ✱

Tangent passes through $(\sqrt{a^2+b^2},0)$:

$\Rightarrow 0 = m\sqrt{a^2+b^2}+c \Rightarrow c^2=m^2(a^2+b^2)$

$\therefore a^2m^2+b^2=m^2a^2+m^2b^2$

$[\because b\neq 0]$: $m^2=1 \Rightarrow m=\pm 1$ ✱

Since graph is symmetrical about the axes: tangents pass through $(-\sqrt{a^2+b^2},0)$ also has gradient ± 1.

$\Rightarrow$ vertices of square are $(\pm\sqrt{a^2+b^2},0)$ and ✱

6. $f: x \to \frac{3x+2}{x-3}$

(a) $f^2(x) = \frac{3(\frac{3x+2}{x-3})+2}{(\frac{3x+2}{x-3})-3} = x$

$\therefore f^2: x\to x,\ x\neq 3$ ✱

$\therefore f^{-1}: x \to \frac{3x+2}{x-3},\ x\neq 3,\ (f=f^{-1})$ ✱

(b) $f^{17}(4) = f[f^2]^8(4) = f(4)$

$= \frac{12+2}{4-3} = 14$ ✱

7. $u_5-u_3 = \lg ar^4 - \lg ar^2 = 2d$

$\therefore d = \lg r$

$u_8-u_5 = \lg(\frac{ar^7}{ar^4}) = 3d \Rightarrow d=\lg r$

$\therefore u_n$ is an AP. ✱

$\sum_{k=n+1}^{2n} u_k = \frac{n}{2}[u_{n+1}+u_{2n}] = \frac{n}{2}[\log ar^n + \log ar^{2n-1}]$

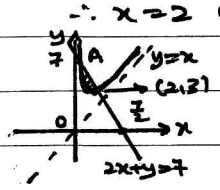
$= \frac{n}{2}[\log a^2r^{3n-1}]$ ✱

8(a) $y=x+\frac{4}{x^2} \Rightarrow y>x$ (since $\frac{4}{x^2}>0$)

$\therefore$ curve lies above $y=x$ ✱

(b) $y'=1-\frac{8}{x^3}$: $y'=0 \Rightarrow x^3=8$

$\therefore x=2$ (only 1 value of x) ✱



For A: $x+\frac{4}{x^2}=7-2x$

$\therefore 3x^3-7x^2+4=0$

$\therefore (x-2)(x-1)(3x+2)=0$

$\therefore A(1,5)$

Area(R) $= \frac{1}{2}(1)(3+5)-\int_1^2(x+\frac{4}{x^2})dx$

$= 4-[\frac{x^2}{2}-\frac{4}{x}]_1^2 = 4-[(2-2)-(\frac{1}{2}-4)] = \frac{1}{2}$ ✱

$V_x = \pi\int_1^2[(7-2x)^2-(x+\frac{4}{x^2})^2]dx$

$= \pi\left[\left[\frac{(7-2x)^3}{3(-2)}\right]_1^2 - \left[\frac{x^3}{3}+8\ln x-\frac{16}{3x^3}\right]_1^2\right]$

$= \pi[\frac{9}{-2}+\frac{125}{6}-(\frac{8-1}{3})-8\ln 2+\frac{16}{3}(\frac{1}{8}-1)]$

$= [\frac{28}{3}-8\ln 2]\pi$ ✱

LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

9. $y=e^{\sqrt{1+x}} \Rightarrow \ln y=\sqrt{1+x}$

$\frac{1}{y}y'=\frac{1}{2\sqrt{1+x}} \Rightarrow 4(1+x)(y')^2=y^2$

$4(1+x)(2)(y')(y'')+4(y')^2=2y(y')$

$\therefore\ 4(1+x)y''+2y'-y=0$ ✱

10. Let $u=e^x+1$: $\frac{du}{dx}=e^x$

$\int_0^1 \frac{e^x}{e^x+1}dx=\int_2^{e+1}\frac{e^x}{u}\left(\frac{du}{e^x}\right)$

$=\int_2^{e+1}\frac{1}{u}du=[\ln u]_2^{e+1}=\ln\left(\frac{e+1}{2}\right)$ ✱

$\int_0^1 \frac{1}{1+e^x}dx=\int_0^1\left[1-\frac{e^x}{1+e^x}\right]dx$

$=[x]_0^1-\ln\left(\frac{e+1}{2}\right)=1-\ln\left(\frac{e+1}{2}\right)$ ✱

11. $y=\frac{4}{x+1}-\frac{1}{x-1} \Rightarrow y'=\frac{-4}{(x+1)^2}+\frac{1}{(x-1)^2}$

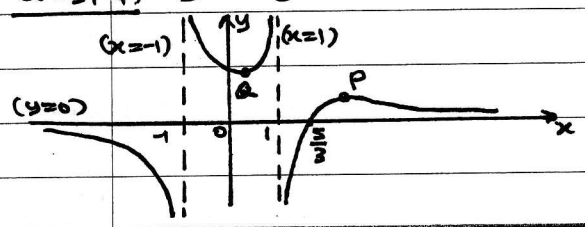
$y''=\frac{8}{(x+1)^3}-\frac{2}{(x-1)^3}$

(a) asymptotes: $y=0,\ x=\pm1$ ✱

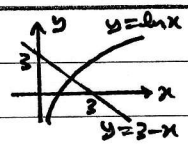
(b) $y'=0$: $\frac{x+1}{x-1}=\pm2 \Rightarrow x=\frac{1}{3}$ or 3

$P(3,\frac{1}{2})$: $y''=-\frac{1}{8}\ (<0) \Rightarrow P$ is max ✱

$Q(\frac{1}{3},\frac{9}{4})$: $y''=\frac{81}{8}\ (>0) \Rightarrow Q$ is min ✱



12. $\ln x+x-3=0 \Rightarrow \ln x=3-x$



$\therefore$ exists only 1 intersection point

$\Rightarrow$ only 1 real root.

Let $f(x)=\ln x+x-3$: $f'(x)=\frac{1}{x}+1$

$f(1)\cdot f(e)=(-2)(e-2)\ (<0)$

$\therefore\ 1<\text{root}(\alpha)<e$ ✱

By NRM: $x_{n+1}=x_n-\frac{\ln x_n+x_n-3}{\frac{1}{x_n}+1}$

$\therefore\ x_{n+1}=\frac{x_n}{1+x_n}\left[(1+x_n)-(\ln x_n+x_n-3)\right]$

$=\frac{x_n(4-\ln x_n)}{1+x_n}$ ✱

Let $x_0=2.2$: $x_1=\frac{2.2(4-\ln 2.2)}{3.2}=2.2079$

$x_2=2.2079$

$\therefore$ root $=2.21$ (3sf) ✱

$a+148°=180° \Rightarrow a=32°$ [Supplementary Angles]

$\theta+d=y$ [Ext. ∡ of Cyclic ABDE]

$\therefore\ \theta=92°-32°=60°=\angle EDA$ ✱

1. $\sin3\theta+\sin5\theta=\cos\theta$

$\therefore\ 2\sin4\theta\cos\theta-\cos\theta=0$

$\cos\theta=0$ or $\sin4\theta=\frac{1}{2}$

$\therefore\ \theta=\frac{\pi}{2},\frac{3\pi}{2}$ or $4\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6},\frac{25\pi}{6},\frac{29\pi}{6},\frac{37\pi}{6},\frac{41\pi}{6}$

$\therefore\ \theta=\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{24},\frac{5\pi}{24},\frac{13\pi}{24},\frac{17\pi}{24},\frac{25\pi}{24},\frac{29\pi}{24},\frac{37\pi}{24},\frac{41\pi}{24}$ ✱

2.(a) $2\cos\theta(\sin3\theta-\sin\theta)$

$=(\sin4\theta+\sin2\theta)-\sin2\theta=\sin4\theta$ ✱

(b) $4(\cos4\theta+\cos2\theta)\sin3\theta\sin\theta$

$=4(\cos4\theta+\cos2\theta)(-\frac{1}{2})(\cos4\theta-\cos2\theta)$

$=2\cos^2 2\theta-2\cos^2 4\theta$

$=(\cos8\theta+1)-(\cos8\theta+1)=\cos4\theta-\cos8\theta$ ✱

(c) $\theta=18°$: [From (a)]

$2\cos18°[\sin54°-\sin18°]=\sin72°=\cos18°$

$\therefore\ \sin54°-\sin18°=\frac{1}{2}$ ✱

(d) $\theta=18°$: [From (b)]

$4[\cos72°+\cos36°]\sin54°\sin18°=\cos72°-\cos144°$

$=\cos72°-(-\cos36°)$

$\therefore\ \sin54°\sin18°=\frac{1}{4}$ ✱

3(a) $\frac{dv}{dt}=(t-1)(v-3) \Rightarrow \int\frac{dv}{v-3}=\int(t-1)dt$

$\ln(v-3)=\frac{1}{2}(t-1)^2+c$

$t=0, v=4$: $c=\frac{1}{2} \Rightarrow \ln(v-3)=\frac{1}{2}[(t-1)^2+1]$

$\therefore\ v=3+e^{\frac{1}{2}[(t-1)^2+1]}$ ✱

[since $e^{\frac{1}{2}[(t-1)^2+1]}>0,\ \forall t\in\mathbb{R}$] $\Rightarrow v>0$ ✱

(b) $\frac{dx}{dt}=t^2e^{-3x} \Rightarrow \int e^{3x}dx=\int t^2dt$

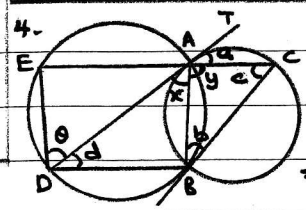
$\therefore\ \frac{1}{3}e^{3x}=\frac{1}{3}t^3+c$

$t=0$: $\frac{1}{3}e^{0.75}=c \Rightarrow e^{3x}=t^3+e^{0.75}$

$\therefore\ x=\frac{1}{3}\ln[t^3+e^{0.75}]$ ✱

$t=0.5$: $x=\frac{1}{3}\ln[\frac{1}{8}+e^{0.75}]=0.2691$ tonne ✱

4.



(a) $b=d$ [∡ of alternate segment]

$b=a$ [∡ of alternate segment]

$\therefore\ a=d$ [equal corresponding ∡]

$\Rightarrow EC // DB$ ✱

(b) $x=c$ [∡ of alternate segment]

$x+y=\angle EAT$ [opposite ∡]

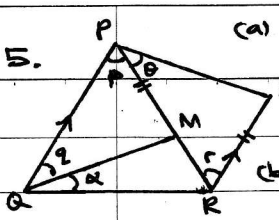
$\therefore\ y=148°-56°=92°=\angle BAC$ ✱

LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK Chung Hwa, Kota Bahru (2006)

No.: ..... MT2 Date: .....

5.



(a) PM = RS (given)
p = r (alternate ∠, QP // RS)
PQ = RP (ΔPQR equilateral)
∴ ΔPQM ≡ ΔRPS [SAS] #

(b) α = 21° : q + α = 60° [✡]
∴ q = 39°

∠RPS = θ = q = 39° [congruent Δ] #

6. $\underline{V}_{AB} = \begin{pmatrix}30\\40\end{pmatrix} - \begin{pmatrix}15\\12\end{pmatrix} = \begin{pmatrix}15\\28\end{pmatrix}$

$= \sqrt{15^2+28^2}$ kmh^{-1}, to N $\tan^{-1}\left(\frac{15}{28}\right)$E

= 31.76 kmh^{-1}, to N 28.2° E #

For nearest distance: $\left[\begin{pmatrix}-5\\-12\end{pmatrix} + t\begin{pmatrix}15\\28\end{pmatrix}\right]\cdot\begin{pmatrix}15\\28\end{pmatrix} = 0$

∴ $-75 - 336 + t(1009) = 0 \Rightarrow t = \frac{411}{1009}$ hour

∴ $SD = \left|\begin{pmatrix}-5\\-12\end{pmatrix} + \frac{411}{1009}\begin{pmatrix}15\\28\end{pmatrix}\right| = \left|\begin{pmatrix}1.110\\-0.595\end{pmatrix}\right|$

$= \sqrt{1.110^2 + (-0.595)^2} = 1.26$ km #

To intercept: $\underline{V}_{AB}$ (to West)

∴ $50\sin\theta = 12 \Rightarrow \theta = 13.9°$

∴ Direction = to N 76.1° W #

When A is at East of B:

distance $= -5 + \frac{12}{28}(15) = \frac{10}{7}$ km

∴ time taken $= \left(\frac{10}{7} \Big/ 12 + 50\cos 13.9°\right)$ h

= 1.4 minutes #

7. $X+Y \sim N(20+\mu, 25+\sigma^2)$

$X-Y \sim N(20-\mu, 25+\sigma^2)$

$P(X+Y>75) = 0.06 \Rightarrow \frac{75-20-\mu}{\sqrt{25+\sigma^2}} = z_{0.06}$

∴ $55 - \mu = 1.555\sqrt{25+\sigma^2}$ — ①

$P(X-Y>10) = 0.02 \Rightarrow \frac{10-20+\mu}{\sqrt{25+\sigma^2}} = z_{0.02}$

∴ $\mu - 10 = 2.054\sqrt{25+\sigma^2}$ — ②

① + ② : $45 = 3.609\sqrt{25+\sigma^2}$

∴ $\sqrt{25+\sigma^2} = 12.47 \Rightarrow \sigma = 11.4$ #

② : $\mu = 10 + 2.054(12.47) = 35.6$ #

8.

	1	2	3
1 (3/6)	2 (9/36)	3 (6/36)	4 (3/36)
2 (3/6)	3 (6/36)	4 (4/36)	5 (2/36)
3 (1/6)	4 (3/36)	5 (2/36)	6 (1/36)

$P_2 = \frac{9}{36}$ $P_3 = \frac{12}{36}$
$P_4 = \frac{10}{36}$ $P_5 = \frac{4}{36}$
$P_6 = \frac{1}{36}$ #

$E(X) = \frac{1}{36}[2(9)+3(12)+4(10)+5(4)+6(1)] = \frac{10}{3}$ #

$E(X^2) = \frac{1}{36}[2^2(9)+3^2(12)+4^2(10)+5^2(4)+6^2(1)] = \frac{110}{9}$

∴ $Var(X) = \frac{110}{9} - \left(\frac{10}{3}\right)^2 = \frac{10}{9}$ #

9. X = no. of draws

$X \sim B(120, 0.42) \rightarrow X \sim N(50.4, 29.232)$

$P(X \geq 45) = P(X > 44.5)$

$= P\left(Z > \frac{44.5-50.4}{\sqrt{29.232}}\right) = 0.86242$ #

10.(a) m = x = RM4.52 #

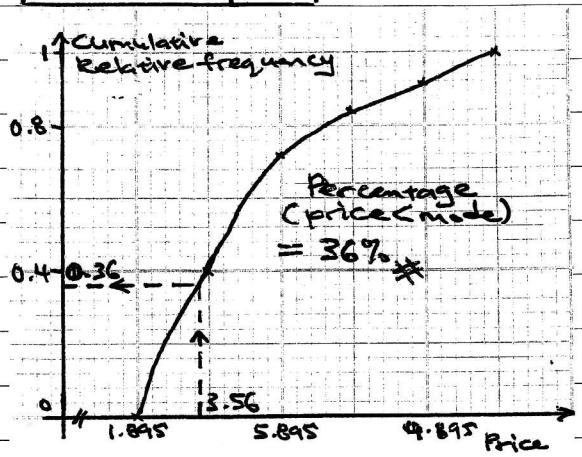
IQR $= \frac{1}{2}[6.00+6.90] - \frac{1}{2}[2.84+2.94] = 3.56$ #

(b)

Price	f
1.895 ≤ x < 3.895	10
3.895 ≤ x < 5.895	8
5.895 ≤ x < 7.895	3
7.895 ≤ x < 9.895	2
9.895 ≤ x < 11.895	2

mode $= 1.895 + \left(\frac{10}{10+2}\right)(2)$

= RM 3.56



11. F(a) = 1 : $a - \frac{1}{4}a^2 = 1 \Rightarrow a^2 - 4a + 4 = 0$

∴ $(a-2)^2 = 0 \Rightarrow a = 2$ #

∴ $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} 1 - \frac{1}{2}x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$ #

$P(Y<y) = P(1 - \frac{X}{2} < y) = P(X > 2(1-y))$

$= 1 - F(2-2y) = 1 - (2-2y) + \frac{1}{4}(2-2y)^2$

$= y^2$ for $0 \leq 2-2y \leq 2$

∴ $P(Y<y) = y^2$, for $0 \leq y \leq 1$ #

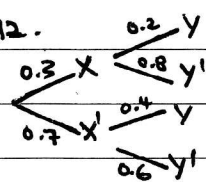
$g(y) = \frac{d}{dy}[P(Y<y)] = \begin{cases} 2y & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$ #

$E(X) = \int_0^2 (x - \frac{1}{2}x^2)\,dx = \left[\frac{1}{2}x^2 - \frac{1}{6}x^3\right]_0^2$

$= \frac{1}{2}(4-0) - \frac{1}{6}(8-0) = \frac{2}{3}$

$E(Y) = \int_0^1 2y^2\,dy = \frac{2}{3}[y^3]_0^1 = \frac{2}{3} = E(X)$ #

12.



$P(X \cap Y) = (0.3)(0.2) = 0.06$ #

$P(X \cup Y) = 1 - 0.7(0.6)$

$= 0.58$ #