LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK CHUNG LING BW (2006)
MATHEMATICS T (PAPER 1)

1. Given that $y = \log_a x^3$ and $z = \log_x a$, show that $yz = 3$. [2]
   Hence, find the values of y and z if $\log_a(3\log_a x) - \log_a(\log_x a) = \log_a 27$. [6]

2. Complete the square for the polynomial $px^2 - x + q$. Hence, show that the roots of the quadratic equations $px^2 - x + q = 0$, $p \neq 0$ is given by $x = \frac{1}{2p}(1 \pm \sqrt{1-4pq})$. [5]
   Deduce the condition for the roots of the equation $px^2 - x + q = 0$ to be real. [2]
   Determine the possible values of a such that $(a+1)x^2 - x + a - 2 = 0$ has real roots. [3]
   Determine the range of values of a such that the equation $(a+1)x^2 - x + a - 2 = 0$ has real roots which have the same sign. [6]

3. Find the sum of the first n terms of the series $\frac{1}{3(4)} + \frac{1}{4(5)} + \frac{1}{5(6)} + \ldots$. [9]

4. Find the set of values of x such that the series $\sum_{n=0}^{\infty}\left(\frac{x}{1+x}\right)^n$ is convergent. [4]
   If the sum to infinity is 4, find the value of x. [2]

5. Expand $(1-x)^{-p}(1-x^2)^{-q}$ as a series in ascending powers of x, up to and including the term in x^3. [6]
   (a) If the first three terms are $1 - 2x + \frac{10}{9}x^2$, find the values of p and q. [3]
   (b) If $q = -\frac{1}{8}$ and the value of p is such that the coefficient of x^2 has the minimum value, show that the coefficient of x^3 is zero. [5]

6. Given $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$, find the values of m and n such that $\mathbf{A}^2 + m\mathbf{A} + n\mathbf{I} = 0$. [3]
   Hence, deduce $\mathbf{A}^3$ and $\mathbf{A}^{-1}$. [7]

7. Find the equation of the tangent to the rectangular hyperbola $xy = c^2$ at the point $P(cp, \frac{c}{p})$. [5]
   By assuming $p > 1$, find in terms of p and c, the perpendicular distance of the origin O from the tangent. [2]

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8. The function f is defined by $f(x) = |x-3|$, $x \in \mathcal{R}$.
   Determine if $f(x)$ is continuous at $x = 3$. Sketch the graph of the function f. [7]

9. The parametric equations of a curve are $x = \frac{2}{t-3}$ and $y = \frac{t}{t-3}$. Show that $\frac{dy}{dx} = \frac{3}{2}$. [4]

10. If $y = \sqrt{x(x-1)(x-3)}$, show that $2y\frac{dy}{dx} = 3x^2 - 8x + 3$ and $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3x - 4$. [4]

11. Evaluate $\int_2^9 \frac{x^2+x-1}{x^2-1}\,dx$, correct to 3 significant figures. [5]

12. Find the coordinates of the points of intersection of the curves $y = x^2$ and $y = 2 - x^2$. [3]
    Find the area bounded by these two curves. [3]
    The region bounded by these two curves is rotated about the y–axis through an angle of π. Find the volume of the solid generated. [4]

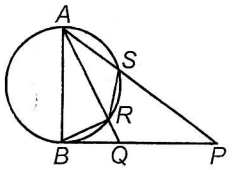
SMJK CHUNG LING BW (2006)
MATHEMATICS T (PAPER 2)

1. $OABC$ is a parallelogram. D is on OA such that $AD : DO = 2 : 1$, E is the midpoint of OC, and F is the intersection point of CD with BE.
   (a) If $CF : CD = k : 1$, show that $\overrightarrow{CF} = k(\frac{1}{3}\overrightarrow{OA} - \overrightarrow{OC})$. [3]
   (b) If $EF : EB = \ell : 1$, prove that $\overrightarrow{EF} = \ell(\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC})$. [3]
   (c) Hence, find k and ℓ. [6]

2. Express $2\sin x + \cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
   Find the maximum and minimum values of $2\sin x + \cos x$ and the corresponding values of x for $0 < x < 2\pi$.
   Hence, solve $\cos x(\cos x + 2\sin x) = 1$ for $0 < x < 2\pi$, correct to 2 decimal places. [2]

3. Prove that $\frac{\cos x - 2\cos 3x + \cos 5x}{\cos x + 2\cos 3x + \cos 5x} = -\tan^2 x$. [5]

4. AB is a diameter of the circle, and BP is a tangent to the circle at B. Prove that



(a) $\angle PQR + \angle PSR = 180^\circ$, [4]

(b) $AS.AP = AR.AQ$. [5]

5. The variable x and y are related by the differential equation $\frac{dy}{dx} = \frac{2x+y+2}{2x+y-1}$.

Using the substitution $z = 2x + y$, show that $\frac{dz}{dx} = \frac{3z}{z-1}$. [3]

Hence, find the general solution. [4]

6. The rate of change of water temperature is given by the differential equation $\frac{d\theta}{dt} = -k(\theta - \theta_s)$, where θ is the temperature of the water at time t, θ_s is the surrounding temperature and k is a positive constant. Water boiled to a temperature of 100°C and is then allowed to cool in a kitchen which has a room temperature of 20°C. It takes the water 1 hour to cool down to a temperature of 80°C.

Show that $k = \ln\frac{4}{3}$.

When the water reaches a temperature of 40°C, it is placed inside the freezer where the temperature is maintained at –10°C. Find the time needed from the time the water is placed in the freezer until it cool down to 0°C. [8]

7. The bus service in a certain town is provided by three companies X, Y and Z in percentage 20%, 45% and 35% respectively. The probabilities that a bus from companies X, Y and Z is air-conditioned are 0.8, 0.7 and 0.6 respectively. A bus is selected at random. Find the probability that

(a) the bus is air-conditioned, [2]

(b) the bus is from company Y given that the bus is air-conditioned. [3]

8. The table shows the probability of the successful firing of randomly chosen emergency flares of various ages.

Age (years)	1	5	10
Probability	0.996	0.970	0.750

(a) Using a suitable approximation, find the probability that of 1000 randomly chosen 1 year old flares, at most 4 fail to fire successfully. [4]

(b) Find the probability that, of 6 randomly chosen 10-years old flares, at least 3 fire successfully. [4]

9. In the production of compact discs at a certain factory the proportion of faulty discs is known to be 0.2. Each week the factory produces 2000 discs. Find the probability that there will be at most 349 faulty discs produced in one week. [4]

10. The continuous random variable X is defined for $0 \le x \le 4$ such that

$$P(X > x) = \begin{cases} 1 - \alpha x & , 0 \le x \le 1 \\ \beta - \frac{x}{6} & , 1 < x \le 4 \end{cases}, \text{ where } \alpha \text{ and } \beta \text{ are constants.}$$

(a) Show that $\alpha = \frac{1}{2}$ and $\beta = \frac{2}{3}$. [2]

(b) Find the cumulative distribution function of X. [2]

(c) Find the probability density function of X. [2]

(d) Determine the expected value and standard deviation of X. [6]

11. The discrete random variable X has $E(X) = 2.6$ and $E(X^2) = 7.0$. Evaluate $E[(X-3)^2]$. [4]

12. In an agricultural experiment, the increase in the masses of 100 sheep in a certain period are recorded in the table.

Increase in mass (kg)	Frequency
4 – 8	3
9 – 13	8
14 – 18	25
19 – 23	34
24 – 28	15
29 – 33	12
34 – 38	3

(a) Calculate the mean and the standard deviation of the increase in the mass of the sheep. [5]

(b) Plot a cumulative frequency curve of the above data.

Hence, estimate the median and the semi interquartile range. [9]

(c) Find the percentage of the sheep with masses is at least 30 kg. [3]

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1. $yx = (3\log_a x)\left(\frac{\log_a a}{\log_a x}\right) = 3$ ✱

$\log_a(3\log_a x) - \log_a(\log_x a) = \log_a 27$

$\therefore \log_a\left(\frac{y}{z}\right) = \log_a 27$

$\therefore y\left(\frac{y}{z}\right) = 27 \Rightarrow y = 9$ ✱ $(\because y>0)$

$\therefore z = \frac{3}{9} = \frac{1}{3}$ ✱

2. $px^2 - x + q = p\left(x^2 - \frac{1}{p}x\right) + q$

$= p\left[\left(x - \frac{1}{2p}\right)^2 - \frac{1}{4p^2}\right] + q = p\left(x - \frac{1}{2p}\right)^2 - \frac{1}{4p} + q$ ✱

$px^2 - x + q = 0 \Rightarrow p\left(x - \frac{1}{2p}\right)^2 = \frac{1-4pq}{4p}$

$\therefore x = \frac{1}{2p} \pm \frac{\sqrt{1-4pq}}{2p} = \frac{1 \pm \sqrt{1-4pq}}{2p}$ ✱

For real roots: $1 - 4pq \geq 0 \Rightarrow pq \leq \frac{1}{4}$ ✱

For $(a+1)x^2 - x + a - 2 = 0$ has real roots:

$(a+1)(a-2) \leq \frac{1}{4} \Rightarrow a^2 - a \leq \frac{9}{4}$

$\therefore \left(a - \frac{1}{2}\right)^2 \leq \frac{10}{4} \Rightarrow \left|a - \frac{1}{2}\right| \leq \frac{\sqrt{10}}{2}$

$\therefore \left\{a : \frac{1-\sqrt{10}}{2} \leq a \leq \frac{1+\sqrt{10}}{2}\right\}$ ✱ —①

$\alpha\beta > 0$: $\frac{a-2}{a+1} > 0 \Rightarrow (a-2)(a+1) > 0$

$\therefore a < -1$ or $2 < a$ —②

For real roots with same sign:

$\left\{a : \frac{1-\sqrt{10}}{2} \leq a < -1 \text{ or } 2 < a \leq \frac{1+\sqrt{10}}{2}\right\}$ ✱

3. $\frac{1}{3(4)} + \frac{1}{4(5)} + \frac{1}{5(6)} + \dots + T_n$

$= \sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} = \sum_{r=1}^{n}\left[\frac{1}{r+2} - \frac{1}{r+3}\right]$

$= \left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n+2}\right] - \left[\frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n+2} + \frac{1}{n+3}\right]$

$= \frac{1}{3} - \frac{1}{n+3} = \frac{n}{3(n+3)}$ ✱

4. For convergent: $\left|\frac{x}{1+x}\right| < 1$

$\therefore |x| < |1+x| \Rightarrow x^2 < 1 + 2x + x^2$

$\therefore \left\{x : x > -\frac{1}{2}\right\}$ ✱

$S_\infty = 4$: $\frac{1}{1 - \left(\frac{x}{1+x}\right)} = 4 \Rightarrow 1 - x = 4$

$\therefore x = -3$ ✱

5. $f(x) = (1-x)^{-p}(1-x^2)^{-q}$

$= \left[1 - p(-x) + \frac{(-p)(-p-1)}{2}(-x)^2 + \frac{-p(-p-1)(-p-2)}{6}(-x)^3 + \dots\right]\left[1 - q(-x^2) + \dots\right]$

$= 1 + px + \left[q + \frac{p(p+1)}{2}\right]x^2 + \left[pq + \frac{p(p+1)(p+2)}{6}\right]x^3 + \dots$ ✱

(a) $f(x) = 1 - 2x + \frac{10}{9}x^2 + \dots$: $[x]$: $p = -2$ ✱

$[x^2]$: $q + \frac{(-2)(-1)}{2} = \frac{10}{9} \Rightarrow q = \frac{1}{9}$ ✱

(b) $q = -\frac{1}{8}$: coef of $x^2 = A = \frac{1}{2}(p)(p+1) - \frac{1}{8}$

$\therefore A_{min}$ when $p = \frac{0+(-1)}{2} = -\frac{1}{2}$

$\therefore$ Coef. of $x^3 = -\frac{1}{2}\left(-\frac{1}{8}\right) + \frac{1}{6}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) = 0$ ✱

6. $A^2 + mA + nI = 0$: $\begin{pmatrix} 4 & 0 \\ -5 & 9 \end{pmatrix} + m\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} + n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0$

$\therefore -5 - m = 0 \Rightarrow m = -5$ ✱

$4 + 2m + n = 0 \Rightarrow n = 6$ ✱

$A^2 - 5A + 6I = 0$: $A^3 = 5A^2 - 6A$

$\therefore A^3 = 5\begin{pmatrix} 4 & 0 \\ -5 & 9 \end{pmatrix} - 6\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ -19 & 27 \end{pmatrix}$ ✱

$A^2A^{-1} - 5AA^{-1} + 6IA^{-1} = 0 \Rightarrow A^{-1} = \frac{1}{6}[5I - A]$

$\therefore A^{-1} = \frac{1}{6}\left[\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}\right] = \frac{1}{6}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$ ✱

7. $xy = c^2 \Rightarrow y = \frac{c^2}{x}$ $\quad \therefore y' = -\frac{c^2}{x^2}$

$P\left(cp, \frac{c}{p}\right)$: $m_t = -\frac{c^2}{(cp)^2} = -\frac{1}{p^2}$

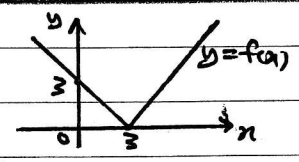
tangent (P): $\frac{y - \frac{c}{p}}{x - cp} = -\frac{1}{p^2} \Rightarrow p^2y + x = 2cp$ ✱

$d_0 = \frac{|0 + 0 + 2cp|}{\sqrt{p^4+1}} = \frac{2cp}{\sqrt{p^4+1}}$ ✱

8. $\lim_{x \to 3^+} f(x) = |3-3| = 0$

$\lim_{x \to 3^-} f(x) = |3-3| = 0$

$f(3) = |3-3| = 0$



$\therefore \lim_{x \to 3^-} f(x) = f(3) = \lim_{x \to 3^+} f(x)$

$\Rightarrow f$ is continuous at $x = 3$ ✱

9. $x = \frac{2}{t-3}$, $y = \frac{t}{t-3} = 1 + \frac{3}{t-3}$

$\therefore y = 1 + \frac{3}{2}x \Rightarrow \frac{dy}{dx} = \frac{3}{2}$ ✱

10(a) $y = \sqrt{x(x-1)(x-3)} \Rightarrow y^2 = x^3 - 4x^2 + 3x$

$\therefore 2y\frac{dy}{dx} = 3x^2 - 8x + 3$ ✱

(b) $2yy'' + 2y'(y') = 6x - 8$

$\therefore y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3x - 4$ ✱

11. $\int_2^9 \frac{x^2+x-1}{x^2-1}\,dx = \int_2^9 \left[1 + \frac{x}{x^2-1}\right]dx$

$= \left[x + \frac{1}{2}\ln(x^2-1)\right]_2^9 = (9-2) + \frac{1}{2}\ln\left[\frac{81-1}{4-1}\right]$

$= 7 + \frac{1}{2}\ln\left(\frac{80}{3}\right) = 8.64$ (3sf) ✱

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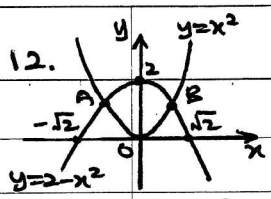
12.



$\left.\begin{array}{l} y=x^2 \\ y=2-x^2 \end{array}\right\}\ x^2=1 \Rightarrow x=\pm 1$

$\therefore A(-1,1),\ B(1,1)$ ✱

$\text{Area} = 2\int_0^1 (2-x^2-x^2)\,dx$

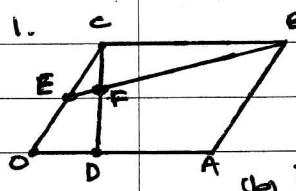
$= 2\left[2x-\frac{2}{3}x^3\right]_0^1 = 2\left(2-\frac{2}{3}\right) = \frac{8}{3}$ ✱

$V_y = \pi\int_0^1 y\,dy + \pi\int_1^2 (2-y)\,dy$

$= \frac{\pi}{2}\left[y^2\right]_0^1 + \frac{\pi}{-2}\left[(2-y)^2\right]_1^2$

$= \frac{\pi}{2}[1-0] - \frac{\pi}{2}[0-1] = \pi$ ✱

1.



(a) $\overrightarrow{CF} = k\,\overrightarrow{CD}$

$= k[\overrightarrow{OD} - \overrightarrow{OC}]$

$= k\left(\frac{1}{3}\overrightarrow{OA} - \overrightarrow{OC}\right]$ ✱

(b) $\overrightarrow{EF} = l\,\overrightarrow{EB} = l(\overrightarrow{EC}+\overrightarrow{CB})$

$\therefore \overrightarrow{EF} = l\left[\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC}\right]$ ✱

(c) $\overrightarrow{EF} = \overrightarrow{EC} + \overrightarrow{CF}$:

$\therefore l\left[\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC}\right] = \frac{1}{2}\overrightarrow{OC} + k\left[\frac{1}{3}\overrightarrow{OA} - \overrightarrow{OC}\right]$

$\therefore l = \frac{k}{3}$ and $\frac{l}{2} = \frac{1}{2} - k$

$\therefore \frac{1}{6}k = \frac{1}{2} - k \Rightarrow k = \frac{3}{7},\ l = \frac{1}{7}$ ✱

2. $2\sin x + \cos x = R\sin(x+\alpha) = f(x)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$\therefore \left.\begin{array}{l} R\cos\alpha = 2 \\ R\sin\alpha = 1 \end{array}\right\}\ \tan\alpha = \frac{1}{2} \Rightarrow \alpha = 0.464$

$R\left(\frac{1}{\sqrt{5}}\right) = 1 \Rightarrow R = \sqrt{5}$

$\therefore f(x) = \sqrt{5}\sin(x+0.464)$ ✱

$\therefore f(x)_{max} = \sqrt{5}(1)$, when $x + 0.464 = \frac{\pi}{2}$

$= \sqrt{5}$, when $x = 1.107$ ✱

$\therefore f(x)_{min} = \sqrt{5}(-1)$, when $x = \frac{3\pi}{2} - 0.464$

$= -\sqrt{5}$, when $x = 4.248$ ✱

$\cos x(\cos x + 2\sin x) = 1$

$\sqrt{5}\sin(x+0.464)\cos x = 1$

$\frac{\sqrt{5}}{2}\left[\sin(2x+0.464) + \sin 0.464\right] = 1$

$\therefore \sin(2x+0.464) = \frac{2}{\sqrt{5}} - \sin 0.464$

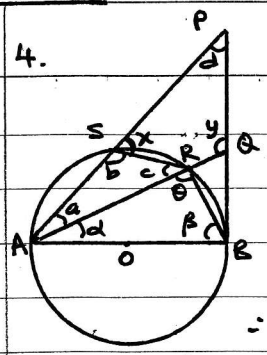
$\therefore 2x + 0.464 = 0.464,\ 2.678,\ 2\pi + 0.464,\ 2\pi + 2.678,\ 4\pi + 0.464$

$\therefore x = 0,\ \pi,\ 2\pi,\ 1.107,\ 4.248$ ✱

3. $\frac{\cos x - 2\cos 3x + \cos 5x}{\cos x + 2\cos 3x + \cos 5x} = \frac{2\cos 3x\cos 2x - 2\cos 3x}{2\cos 3x\cos 2x - 2\cos 3x}$

$= \frac{\cos 2x - 1}{\cos 2x + 1} = \frac{-2\sin^2 x}{2\cos^2 x} = -\tan^2 x$ ✱

4.



(a) $x = \beta$ [ext ∠ of cyclic ABRS]

$\theta = 90^\circ$ [∠ in semicircle]

$\alpha + \beta = 90^\circ$ [complementary ∠]

$y = \alpha + 90^\circ$ [Ext. ∠ of △ and OB tangent]

$\therefore y + \alpha + \beta = 180^\circ + \alpha$

$\therefore y + x = 180^\circ = \angle PQR + \angle PSR$ ✱

(b) $b + x = 180^\circ$ [supplementary ∠]

$\therefore b = y$

a is a common angle of $\triangle SAR, \triangle QAP$

$c = d$ [ext. ∠ of cyclic RSPQ, $x+y=180^\circ$]

$\therefore \triangle SAR \sim \triangle QAP \Rightarrow \frac{AS}{AQ} = \frac{AR}{AP}$

$\therefore AS.AP = AR.AQ$ ✱

5. $z = 2x + y$: $\frac{dz}{dx} = 2 + \frac{dy}{dx}$

$\therefore \frac{dz}{dx} = 2 + \frac{z+2}{z-1} = \frac{3z}{z-1}$ ✱

$\therefore \int\left[1 - \frac{1}{z}\right]dz = \int 3\,dx \Rightarrow z - \ln z = 3x + c$

$\therefore \ln(2x+y) = y - x - c$ ✱

6. $\frac{d\theta}{dt} = -k(\theta - 20)$: $\int \frac{d\theta}{\theta - 20} = \int -k\,dt$

$\therefore \ln(\theta - 20) = -kt + c$

$t=0$: $\ln 80 = c \Rightarrow \ln\left[\frac{\theta-20}{80}\right] = -kt$

$t=1$: $\ln\left(\frac{60}{80}\right) = -k \Rightarrow k = -\left[-\ln\left(\frac{4}{3}\right)\right] = \ln\frac{4}{3}$ ✱

$\frac{d\theta}{dt} = -k(\theta + 10)$: $\int \frac{d\theta}{\theta+10} = \int -k\,dt$

$\therefore \ln(\theta + 10) = -kt + h$

$t=0$: $\ln 50 = h \Rightarrow \ln\left[\frac{\theta+10}{50}\right] = -kt$

$\theta = 0$: $t = \ln\left(\frac{10}{50}\right) / -\ln\left(\frac{4}{3}\right) = 5.595$ hour ✱

7. (a) $P(AC) = 0.2(0.8) + 0.45(0.7) + 0.35(0.6)$

$= 0.685$ ✱

(b) $P(Y|AC) = \frac{0.45(0.7)}{0.685} = \frac{67}{137}$ ✱

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8. (a) X = no. of unsuccessful flares

$X \sim B(1000, 0.004) \rightarrow X \sim P_o(4)$

$P(X \le 4) = e^{-4}\left[1+4+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}\right]$

$= 0.6288$ #

(b) Y = no. of successful flares

$Y \sim B(6, 0.75)$

$P(Y \ge 3) = 1-\left[0.25^6 + 6(0.25)^5(0.75) + 15(0.25)^4(0.75)^2\right] = 0.9624$ #

9. X = no. of faulty discs

$X \sim B(2000, 0.2) \rightarrow X \sim N(400, 320)$

$P(X \le 349) = P(X < 349.5)$

$= P\left(Z < \frac{349.5-400}{\sqrt{320}}\right) = 0.00238$ #

10. (a) $x=4$: $\beta - \frac{4}{6} = 0 \Rightarrow \beta = \frac{2}{3}$ #

$x=1$: $1-\alpha = \beta - \frac{1}{6} \Rightarrow \alpha = \frac{1}{2}$ #

(b) $F(x) = P(X \le x)$

$= 1 - P(X > x) = \begin{cases} 0 & x<0 \\ \frac{1}{2}x & 0 \le x \le 1 \\ \frac{1}{3}+\frac{x}{6} & 1 < x \le 4 \\ 1 & 4 < x \end{cases}$ #

(c) $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} \frac{1}{2} & 0 \le x \le 1 \\ \frac{1}{6} & 1 < x \le 4 \\ 0 & \text{otherwise} \end{cases}$ #

(d) $E(X) = \int_0^1 \frac{1}{2}x\,dx + \int_1^4 \frac{1}{6}x\,dx$

$= \frac{1}{4}[x^2]_0^1 + \frac{1}{12}[x^2]_1^4$

$= \frac{1}{4}(1-0) + \frac{1}{12}(16-1) = 1.5$ #

$E(X^2) = \int_0^1 \frac{1}{2}x^2\,dx + \int_1^4 \frac{1}{6}x^2\,dx$

$= \frac{1}{6}[x^3]_0^1 + \frac{1}{18}[x^3]_1^4$

$= \frac{1}{6}(1-0) + \frac{1}{18}(64-1) = \frac{11}{3}$

$\therefore s = \left[\frac{11}{3} - \left(\frac{3}{2}\right)^2\right]^{1/2} = \left[\frac{17}{12}\right]^{1/2} = 1.190$ #

11. $E[(X-3)^2] = E[X^2 - 6X + 9]$

$= 7 - 6(2.6) + 9 = 0.4$ #

12(a)

x	f
6	2
11	8
16	25
21	34
26	15
31	12
36	3

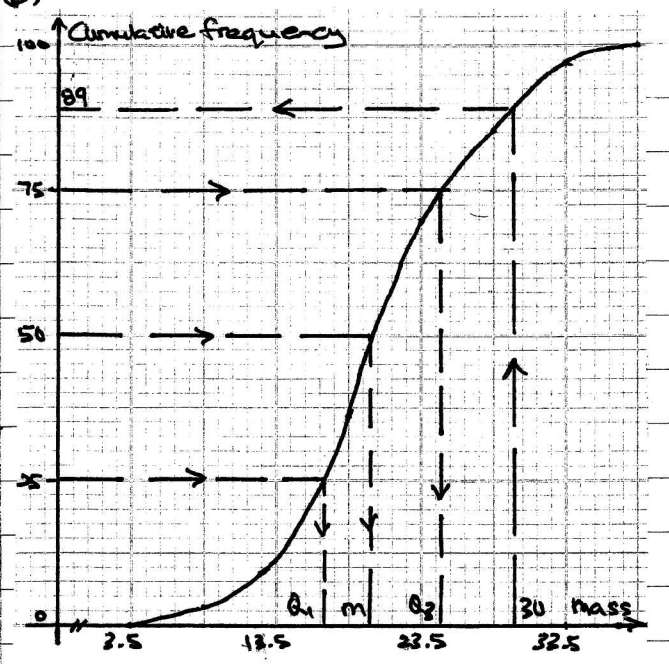
$n = 100$.

$\bar{x} = \frac{2090}{100} = 20.9$ #

$s = \left[\frac{48020}{100} - 20.9^2\right]^{1/2}$

$= 6.595$ #

(b)



$m = 20$ #

$SIQR = \frac{1}{2}[25 - 16.8] = 4.1$ #

(c) Percentage (≥ 30 kg) $= \frac{11}{100} \times 100\%$

$= 11\%$ #

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