LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK CHUNG LING – PG (2006)
MATHEMATICS T (PAPER 1)

1. When the polynomial $x^4 - ax^2 + b$ is divided by $(x+1)^2$, the remainder is $5x - 2$, find the constants a and b. [4]

2. By using the substitution $y = x^2 + 2x$, solve the equation $x^4 + 4x^3 - 8x + 3 = 0$. [4]

3. Find the set of values of x such that $\dfrac{|x+2|+2}{2x-1} > 1$. [5]

4. Let $\mathcal{E}$ denote the universal set and let A, B and C be any three subsets of $\mathcal{E}$. Simplify each of the following using the laws of algebra of sets :

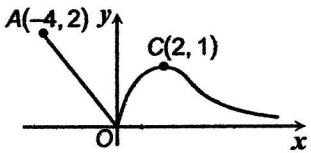
   (a) $[A \cap (B' \cap C)'] \cup (C' \cup B)$ (b) $[A \cap (C - A')] - (C' - A)$ [6]

5. (a) Write down the first three terms in the expansion of $\sqrt{x + \frac{2}{x}}$ in descending powers of x. State the range of x for the expansion to be valid.

   Hence, evaluate $\sqrt{4.5}$, correct to 4 decimal places. [6]

   (b) Find $\displaystyle\sum_{r=3}^{n}\left[3n - \frac{1}{(2r+1)(2r+3)} + \log_a 2a^r\right]$ in terms of n. [5]

6. The diagram shows the graph $y = f(x)$. There is a maximum at the point $C(2, 1)$. Sketch, on separate diagrams, the graphs of



   (a) $y = -f(x) + 2$

   (b) $y = f(-2x)$

   (c) $y = -f(2 - x)$

   Labelling each graph clearly and showing the coordinates of the points corresponding to A, B and C. [7]

7. A circle with radius r and centre in the forth quadrant $(x > 0, y < 0)$ touches both the axes. This circle also touches the line $3x - 4y - 12 = 0$.

   Find the two values of r and obtain the equations of the two corresponding circles.

   Find the equation of the forth tangent to the two circles. [6]

8. The functions f and g are defined as follows :

   $$f : x \to x^2 + 3x + 4,\ x \in S, \text{ and } g : x \to \ln x,\ 0 < x \le 1,$$

   where function f is one–one and increasing.

   (a) Sketch the graph of f and hence find the largest S. [3]

   (b) Define f^{-1} explicitly, stating its domain and range. Sketch the graph of f^{-1} in the same diagram as f. [5]

   (c) Determine, with reason, the existence of gof.

   Given that fog exists, show that $fog(x) = 0$ has exactly one root. [4]

9. (a) Find $\displaystyle\int \frac{x+1}{(x+3)(x-1)}\,dx$. [3]

   (b) Evaluate $\displaystyle\int_0^1 x^3 e^{-2x}\,dx$. [5]

   (c) Find an approximate value of $\displaystyle\int_0^{0.4} \ln(3 + x)\,dx$ by using the trapezium rule with five ordinates. Give your answer correct to 3 decimal places.

   Sketch the curve and determine whether the estimated value is larger or smaller than the exact value. [5]

10. (a) Given that matrix P is given by $\begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & 0 \\ -1 & -3 & 1 \end{pmatrix}$. Find the real numbers s and t such that $P^2 = sP + tI$, where I is the 3×3 identity matrix.

    Deduce that $P^4 = -5P + 6I$. [4]

    (b) There are three commodities X, Y and Z which are bought and sold by three dealers A, B and C. Dealer A purchases 2 units of X and 5 units Z and sells 3 units of Y; dealer B purchases 5 units of X, 2 units of Y and sells 7 units of Z; and another dealer C purchases 3 units of Y, 1 unit of Z and sells 4 units of X. In the process A earns RM 11 and C earns RM 5 but B loses RM 12. If the earning while purchasing equals the losing while selling for each respective units find the earning of each of the commodities X, Y and Z. [6]

11. (a) Obtain the three roots z_1, z_2 and z_3 of the equation $z^3 - 8 = 0$.

    Find the modulus and argument of z_1, z_2 and z_3. [5]

    (b) Given that $|z - 3 + 3i| = 4$. Sketch the locus for z.

    Hence find the least and greatest values of

    (i) $|z + 1|$ (ii) $\arg(z + 1)$ [6]

12. P is a variable point on the rectangular hyperbola. The tangent at P cuts the x–axis and y–axis at A and B respectively. Q is the fourth vertex of the parallelogram $OABQ$. Show that as P moves along the curve

    (a) $xy = c^2$, Q also describes a hyperbola with the same asymptotes.

    (b) $x^2 - y^2 = a^2$, then the equation of the locus of Q is $a^2(y^2 - x^2) = x^2y^2$. [11]

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MATHEMATICS T (PAPER 2)

1. Express $5\sin\theta + 12\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and α is acute, giving the value of α to the nearest 0.1°. [3]
   Hence solve the equation $6\sec\theta - 5\tan\theta = 12$ for $0^\circ < \theta < 360^\circ$. [4]

2. In an acute-angled triangle ABC, the lengths of the sides BC, CA and AB are denoted by a, b and c respectively. The $\angle BAC$ is denoted by A and N is the foot of the perpendicular from C to AB. By considering appropriate triangles, show that $a^2 = (b\sin A)^2 + (c - b\cos A)^2$ and deduce the cosine formula. [4]

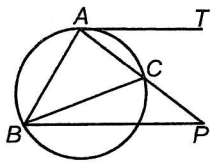
   Express $\cos A$ in terms of a, b and c, and hence show that
   $$\cos^2\left(\frac{A}{2}\right) = \frac{1}{4bc}(b + c - a)(b + c + a).$$ [4]

3. Use the substitution $y = u - 2x$ to find the general solution of the differential equation $\frac{dy}{dx} = -\frac{8x + 4y + 1}{4x + 2y + 1}$. [5]

4. In a model of mortgage repayment, the sum of money owed to a bank is denoted by x and the time is denoted by t. The sum of money owed to the bank due to interest, increases at a rate proportional to the sum of the money owed. Money is also repaid at a constant rate r.
   When $x = a$, interest and repayment balance. Show that $\frac{dx}{dt} = \frac{r}{a}(x - a)$. [3]
   Given that, when $t = 0$, $x = A$. Find x in terms of t, r, a and A (for $A < a$). [5]
   If the loan is repaid in a finite time, T, show that $T = \frac{r}{a}\ln\left(\frac{a}{a - A}\right)$. [3]

5. (a) Four boats A, B, C and D are travelling with constant velocities. The velocity of B relative to A is $10\underline{i} + 10\underline{j}$, the velocity of B relative to C is $20\underline{i} - 5\underline{j}$ and the velocity of D relative to C is $-14\underline{i} - 5\underline{j}$. Find the velocity of A relative to D. [5]
   (b) A boy cycling with velocity $u\underline{i}$ feels that the wind is as blowing with velocity $\frac{1}{2}v_1(\underline{i} - \sqrt{3}\,\underline{j})$. But when he alters his velocity to $\frac{1}{2}u(-\sqrt{3}\,\underline{i} + \underline{j})$, the wind is as blowing with velocity $v_2\underline{i}$.
   Find the actual velocity of the wind and the value of v_2 in terms of u. [7]

6. AT is a tangent to the circle at A. BP is parallel to AT and ACP is a straight line. Show that $AP.\,AC = AB^2$. [7]



7. Find the number of 4–letter code–words that can be formed from the letters of the words *ADVANCE*,
   (a) using neither of the "*A*"s, (b) using both of the "*A*"s. [5]

8. A multiple choice examination consists of 20 questions each of which the candidate is required to tick as correct one of 3 possible answers. Exactly one answer to each question is correct. A correct answer gets 1 mark and a wrong answer get 0 marks. Consider a candidate who has complete ignorance about every question and therefore ticks at random.
   (a) What is the probability that he gets a particular answer correct?
   (b) Calculate the mean and variance of the number of question he answers correctly.
   The examiners wish to ensure that not more than 1% of completely ignorant candidates pass the examination. Using an appropriate approximation, find to the nearest integer, the pass mark that meets this requirement. [8]

9. Serious delays on a certain railway line occur at random, at an average rate of one per week. Show that the probability of at least 4 serious delays occurring during a particular 4–week period is 0.567. [3]
   Taking a year to consist of thirteen 4–week periods, find the probability that, in a particular year, there are at least ten of these 4–week periods during which at least 4 serious delays occur. [4]
   Given that the probability of at least one serious delay occurring in a period of n weeks is greater than 0.995. Find the least possible integer value of n. [3]

10. The random variable X has a normal distribution with mean 3 and variance 4. The random variable S is the sum of 100 independent observations of X, and the random variable T is the sum of another 300 independent observations of X. Giving your answer correct to 3 decimal places, find
    (a) $P(S > 310)$ [3]
    (b) $P(3S > 50 + T)$ [4]
    The random variable Q is the sum of n independent observations of X. Determine the value of $P(Q > 3.5n)$ as n becomes very large, justifying your answer. [4]

11. An author sends his manuscript to a large number of publishers in turn, only approaching each one, after the first, if the one before has refused it. There is a constant probability, p, that his manuscript will be accepted by each publisher approached. The variable M is the number of publishers approached, up to and including the one who accepts the manuscript.
    (a) Write down, in terms of p, the values of $P(M = 1)$, $P(M = 2)$ and $P(M = 3)$. [2]
    (b) Show that M is a discrete random variable. [3]
    Given that $E(M) = \frac{1}{p}$ and $\mathrm{Var}(M) = \frac{1-p}{p^2}$, find $P(|M - E(M)| < \sqrt{\mathrm{Var}(M)})$, for $p = \frac{1}{4}$. [6]

12. 75% of participants in a mathematics quiz are boys. The scores, x, of 30 girls is given by $\sum(x - 72) = 66$, and the scores, y, of boys is given by $\sum(y - 80) = -135$. Find the mean score of
    (a) the girls, (b) all the participants. [3]

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1. Let $f(x) = x^4 - ax^2 + b$: $f'(x) = 4x^3 - 2ax$

$f(x) = (x+1)^2 \cdot p(x) + (5x-2)$

$f'(x) = (x+1)^2 \cdot p'(x) + 2(x+1) \cdot p(x) + 5$

$f'(-1) = -4 + 2a = 0 + 0 + 5 \Rightarrow a = \frac{9}{2}$ ✱

$f(-1) = 1 - \frac{9}{2} + b = 0 - 5 - 2 \Rightarrow b = -\frac{7}{2}$ ✱

2. $y = x^2 + 2x$: $x^4 + 4x^3 - 8x + 3 = 0$

$\therefore (x^4 + 4x^3 + 4x^2) - 4(x^2 + 2x) + 3 = 0$

$\therefore y^2 - 4y + 3 = 0 \Rightarrow (y-1)(y-3) = 0$

$\therefore (x+1)^2 - 1 = 1$ or $(x+1)^2 - 1 = 3$

$\therefore (x+1)^2 = 2$ or $(x+1)^2 = 4$

$\therefore x = -1 \pm \sqrt{2}$ or -1 ± 2

$\therefore x = -1 \pm \sqrt{2}$ or -3 or 1 ✱

3. $\frac{|x+2|+2}{2x-1} > 1 \Rightarrow \frac{|x+2|+3-2x}{2x-1} > 0$

For $x \geq -2$: or For $x < -2$:

$\frac{5-x}{2x-1} > 0$

+	+	−	: $5-x$
−	+	+	: $2x-1$

$\ominus \ \frac{1}{2} \ \oplus \ 5 \ \ominus$: $\frac{5-x}{2x-1}$

$\therefore \frac{1}{2} < x < 5$

$\frac{1-3x}{2x-1} > 0$

+	−	−	: $1-3x$
−	−	+	: $2x-1$

$\ominus \ \frac{1}{3} \ \oplus \ \frac{1}{2} \ \ominus$: $\frac{1-3x}{2x-1}$

$\therefore x = \emptyset$

$\therefore \{x : \frac{1}{2} < x < 5\}$ ✱

4. (a) $[A \cap (B' \cap C)'] \cup (C' \cup B)$

$= [A \cap (B \cup C')] \cup [\Sigma \cap (B \cup C')]$

$= (A \cup \Sigma) \cap (B \cup C') = \Sigma \cap (B \cup C') = B \cup C'$ ✱

(b) $[A \cap (C - A')] - (C' - A)$

$= [A \cap (C \cap A)] \cap (C' \cap A')'$

$= (A \cap A \cap C) \cap (C \cup A) = (A \cap C) \cap (C \cup A)$

$= A \cap [C \cap (C \cup A)] = A \cap [(C \cup \emptyset) \cap (C \cup A)]$

$= A \cap [C \cup (\emptyset \cap A)] = A \cap (C \cup \emptyset) = A \cap C$ ✱

5. (a) $(x + \frac{2}{x})^{1/2} = \sqrt{x}\,(1 + \frac{2}{x^2})^{1/2}$

$= \sqrt{x}\,[1 + \frac{1}{2}(\frac{2}{x^2}) + \frac{1}{2}(\frac{1}{2})(-\frac{1}{2})(\frac{2}{x^2})^2 + \dots]$

$= \sqrt{x} + \frac{1}{x\sqrt{x}} - \frac{1}{2x^3\sqrt{x}} + \dots$ ✱

For Exp. valid: $|\frac{2}{x^2}| < 1 \Rightarrow |x| > \sqrt{2}$

$\{x : x < -\sqrt{2}$ or $\sqrt{2} < x\}$ ✱

Let $x = 4$: $\sqrt{4.5} = 2 + \frac{1}{8} - \frac{1}{256} + \dots$

$= 2.1211$ (4dp) ✱

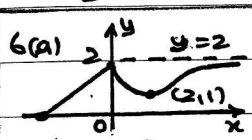
(b) $\sum_{r=3}^{n} [3n - \frac{1}{(2r+1)(2r+3)} + \log_a 2a^r]$

$= \sum_{r=3}^{n} (3n + \log_a + \frac{1}{2}(\frac{1}{2r+3} - \frac{1}{2r+1}) + r]$

$= (n-2)[3n + \log_a 2] + \frac{1}{2}[\frac{1}{2n+3} - \frac{1}{7}] + \frac{(n-2)}{2}(3+n)$

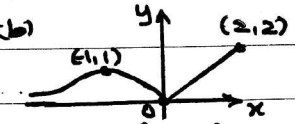
$= \frac{(n-2)}{2}[7n + 2\log_a 2 + 3] + \frac{1}{2}(\frac{1}{2n+3} - \frac{1}{7})$ ✱

6(a)



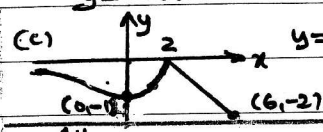
$y = -f(x) + 2$

(b)



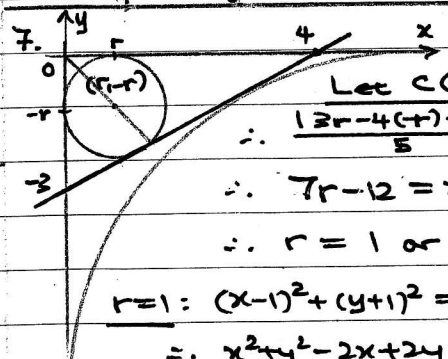
$y = f(-2x)$

(c)



$y = -f(-x+2)$

7.



Let $C(r, -r)$: $d_c = r$

$\therefore \frac{|3r - 4(-r) - 12|}{5} = r$

$\therefore 7r - 12 = \pm 5r$

$\therefore r = 1$ or 6 ✱

$r = 1$: $(x-1)^2 + (y+1)^2 = 1$

$\therefore x^2 + y^2 - 2x + 2y + 1 = 0$ ✱

$r = 6$: $(x-6)^2 + (y+6)^2 = 6$

$\therefore x^2 + y^2 - 12x + 12y + 36 = 0$ ✱

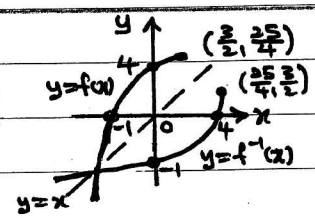
since circles are symmetrical about $y = -x$: (the other tangent)

$4x - 3y - 12 = 0$ ✱

8. (a) $f(x) = -x^2 + 3x + 4$

$= -(x - \frac{3}{2})^2 + \frac{25}{4}$

$S = \{x : x \leq \frac{3}{2}\}$ ✱



(b) $f^{-1}(x) = u$

$\therefore f(u) = x \quad (u \leq \frac{3}{2})$

$\therefore -(u - \frac{3}{2})^2 + \frac{25}{4} = x \Rightarrow u = \frac{3}{2} \pm \sqrt{\frac{25}{4} - x}$

$[\because u \leq \frac{3}{2}]$: $u = \frac{3}{2} - \sqrt{\frac{25}{4} - x}$

$\therefore f^{-1} : x \rightarrow \frac{1}{2}[3 - \sqrt{25 - 4x}]$, $x \leq \frac{25}{4}$ ✱

$D_{f^{-1}} : \{x : x \leq \frac{25}{4}\}$, $R_{f^{-1}} : \{x : x \leq \frac{3}{2}\}$ ✱

(c) since $R_f \not\subset D_g$: gf does not exist. ✱

$fg(x) = 0$: $g(x) = -1 \Rightarrow \ln x = -1$

$\therefore x = e^{-1}$ (only 1 root) ✱

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9.(a) (i) $\int \frac{x+1}{(x+3)(x-1)}dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x-3}dx$

$= \frac{1}{2}\ln(x-1)(x+3) + c$ ✱

(ii) $\int_0^1 x^3 e^{-2x}dx = \left[x^3\left(\frac{e^{-2x}}{-2}\right)\right]_0^1 - \int_0^1 -\frac{3x^2}{2}e^{-2x}dx$

$= \left[-\frac{e^{-2}}{2} - 0\right] + \left[\frac{3x^2}{2}\left(\frac{e^{-2x}}{-2}\right)\right]_0^1 - \int_0^1 -\frac{3x}{2}e^{-2x}dx$

$= -\frac{e^{-2}}{2} + \left[-\frac{3e^{-2}}{4} - 0\right] + \left[\frac{3x}{2}\left(\frac{e^{-2x}}{-2}\right)\right]_0^1 - \int_0^1 -\frac{3}{4}e^{-2x}dx$

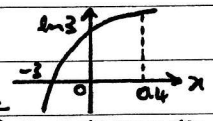
$= -\frac{5e^{-2}}{4} + \left[-\frac{3e^{-2}}{4} - 0\right] - \frac{3}{8}\left[e^{-2x}\right]_0^1$

$= -2e^{-2} - \frac{3}{8}\left[e^{-2} - 1\right] = \frac{1}{8}\left[3 - 19e^{-2}\right]$ ✱

(b) $\int_0^{0.4} \ln(3+x)dx$

$= \frac{1}{2}(0.1)\left[\ln 3 + 2(\ln 3.1 + \ln 3.2 + \ln 3.3) + \ln 3.4\right]$

$= 0.465$ (3dp) ✱



since graph is concave downwards (trapeziums lower than graph) ⇒ estimate value < exact value ✱

10.

(a) $P^2 = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 0 & 1 \\ 1 & 3 & 1 \end{pmatrix} = s\begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & 0 \\ -1 & -3 & 1 \end{pmatrix} + t\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(2,1): $1 = -s \Rightarrow s = -1$ ✱

(1,1): $1 = -1 + t \Rightarrow t = 2$ ✱

$P^4 = (P^2)^2 = (-P+2I)(-P+2I)$

$= P^2 - 2P - 2P + 4I = (-P+2I) - 4P + 4I$

$\therefore P^4 = -5P + 6I$ ✱

(b) Let x, y, z are earnings of each of x, y and z respectively.

$\left.\begin{array}{l} 2x - 3y + 5z = 11 \\ 5x + 2y - 7z = -12 \\ -4x + 3y + z = 5 \end{array}\right\} \begin{pmatrix} 2 & -3 & 5 \\ 5 & 2 & -7 \\ -4 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -12 \\ 5 \end{pmatrix}$

Let $A = \begin{pmatrix} 2 & -3 & 5 \\ 5 & 2 & -7 \\ -4 & 3 & 1 \end{pmatrix}$:

$\text{adj}(A) = \begin{pmatrix} 23 & 23 & 23 \\ 18 & 22 & 6 \\ 11 & 39 & 19 \end{pmatrix}^T = \begin{pmatrix} 23 & 18 & 11 \\ 23 & 22 & 39 \\ 23 & 6 & 19 \end{pmatrix}$

$|A| = 2(23) + 5(18) + (-4)(11) = 92$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{92}\begin{pmatrix} 23 & 18 & 11 \\ 23 & 22 & 39 \\ 23 & 6 & 19 \end{pmatrix}\begin{pmatrix} 11 \\ -12 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\therefore x = RM1,\ y = RM2,\ z = RM3$ ✱

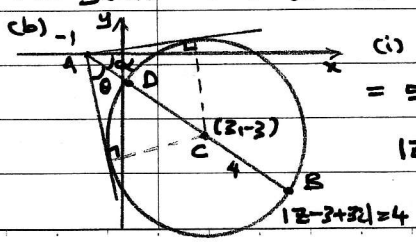
11.(a) $z^3 - 8 = 0 \quad (z-2)(z^2+2z+4) = 0$

$\therefore$ roots $= 2, -1+\sqrt{3}i, -1-\sqrt{3}i$ ✱

$|z_1| = 2 = |z_2| = |z_3|$ ✱

$\therefore \arg(z_1) = 0,\ \arg(z_2) = \frac{2\pi}{3},\ \arg(z_3) = -\frac{2\pi}{3}$ ✱

(b)



(i) $|z+1|_{max} = AB$

$= 5 + 4 = 9$ ✱

$|z+1|_{min} = AD$

$= 5 - 4 = 1$ ✱

(ii) $\theta = \sin^{-1}\left(\frac{4}{5}\right)$, $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

$\arg(z+1)_{min} = -(\theta+\alpha) = -\frac{\pi}{2}$ ✱

$\arg(z+1)_{max} = \theta - \alpha = 0.2838$ ✱

12.(a) $xy = c^2 \Rightarrow \frac{dy}{dx} = -\frac{c^2}{x^2}$

$P(cp, \frac{c}{p})$: $m_T = -\frac{1}{p^2}$; tangent: $\frac{y - \frac{c}{p}}{x - cp} = -\frac{1}{p^2}$

$\therefore p^2y + x = 2cp \quad \therefore A(2cp, 0)$

$\therefore B(0, \frac{2c}{p})$, $Q(x, y)$

$\therefore M_{AQ} = M_{OB}$: $\left(\frac{x+2cp}{2}, \frac{y}{2}\right) = \left(0, \frac{c}{p}\right)$

$\therefore x = -2cp$ and $y = \frac{2c}{p}$

$\therefore xy = -4c^2$ (locus of Q is a hyperbola with asymptotes $x=0, y=0$) ✱

(b) $x^2 - y^2 = a^2$: $\frac{dy}{dx} = \frac{x}{y}$, Let $P(\alpha, \beta)$

$\therefore$ tangent: $\frac{y-\beta}{x-\alpha} = \frac{\alpha}{\beta}$

$\therefore A: (y=0) \quad x = \frac{\alpha^2 - \beta^2}{\alpha} = \frac{a^2}{\alpha}$

$B: (x=0) \quad y = \frac{\beta^2 - \alpha^2}{\beta} = -\frac{a^2}{\beta}$, $Q(x, y)$

$M_{AQ} = M_{OB}$: $x + \frac{a^2}{\alpha} = 0$ and $y = -\frac{a^2}{\beta}$

since $\alpha^2 - \beta^2 = a^2$: $\left(-\frac{a^2}{x}\right)^2 - \left(-\frac{a^2}{y}\right) = a^2$

$\therefore y^2 - x^2 = \frac{x^2y^2}{a^2}$ [locus of Q] ✱

1. $f(\theta) = 5\sin\theta + 12\cos\theta = R\sin(\theta+\alpha)$

$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$\therefore \left.\begin{array}{l} R\cos\alpha = 5 \\ R\sin\alpha = 12 \end{array}\right\}$ $\tan\alpha = 12/5$, $\therefore \alpha = 67.4°$; $R(5/13) = 5$, $\therefore R = 13$

$\therefore f(\theta) = 13\sin(\theta + 67.4°)$ ✱

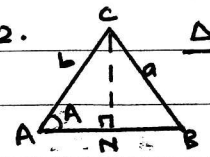
$6\sec\theta - 5\tan\theta = 12 \Rightarrow 5\sin\theta + 12\cos\theta = 6$

$\therefore 13\sin(\theta + 67.4°) = 6$

$\therefore \theta + 67.4° = 152.5°, 387.5°$

$\therefore \theta = 85.1°$ or $320.1°$ ✱

2.



ΔBNC: $a^2 = BN^2 + CN^2$

$= (c - AN)^2 + (b\sin A)^2$

$= (c - b\cos A)^2 + (b\sin A)^2$ ✱

$\therefore a^2 = c^2 - 2bc\cos A + b^2\cos^2 A + b^2\sin^2 A$

$\therefore a^2 = b^2 + c^2 - 2bc\cos A$ ✱ $[\because \cos^2 A + \sin^2 A = 1]$

$\cos A = \frac{b^2+c^2-a^2}{2bc} \Rightarrow 2\cos^2\frac{A}{2} - 1 = \frac{b^2+c^2-a^2}{2bc}$ ✱

$\therefore \cos^2\left(\frac{A}{2}\right) = \frac{b^2+c^2+2bc-a^2}{4bc} = \frac{(b+c)^2-a^2}{4bc}$

$= \frac{(b+c+a)(b+c-a)}{4bc}$ ✱

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3. $y = u - 2x$: $\frac{dy}{dx} = \frac{du}{dx} - 2$

$\frac{dy}{dx} = -\frac{8x+4y+1}{4x+2y+1} \Rightarrow \frac{du}{dx} - 2 = -\frac{4u+1}{2u+1}$

$\therefore \frac{du}{dx} = \frac{4u+2-(4u+1)}{2u+1} = \frac{1}{2u+1}$

$\therefore \int (2u+1)\,du = \int dx \Rightarrow \frac{(2u+1)^2}{4} = x + c$

$\therefore (4x+2y+1)^2 = 4x + A$ ✱, $(A = 4c)$

4. interest: $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$

repaid: $-\frac{dx}{dt} = r$ $\therefore x = a : ka = r \Rightarrow k = \frac{r}{a}$

$\frac{dx}{dt} = \frac{r}{a}x - r = \frac{r}{a}[x - a]$ ✱

$\int \frac{-dx}{a-x} = \int \frac{r}{a}\,dt \Rightarrow \ln(a-x) = \frac{r}{a}t + c$ — ①

$t = 0$: $c = \ln(a - A)$ — ②

$\therefore$ ① − ②: $\ln\left(\frac{a-x}{a-A}\right) = \frac{rt}{a}$

$\therefore \frac{a-x}{a-A} = e^{\frac{rt}{a}} \Rightarrow x = a - (a-A)e^{rt/a}$ ✱

For $x = 0$: $\frac{rT}{a} = \ln\left(\frac{a}{a-A}\right) \Rightarrow T = \frac{a}{r}\ln\left(\frac{a}{a-A}\right)$ ✱

5.(a) $\underline{V}_B - \underline{V}_A = \begin{pmatrix}10\\10\end{pmatrix}$, $\underline{V}_B - \underline{V}_C = \begin{pmatrix}20\\-5\end{pmatrix}$, $V_D - V_C = \begin{pmatrix}-14\\-5\end{pmatrix}$

$V_{AD} = V_A - V_D = \begin{pmatrix}20\\-5\end{pmatrix} - \begin{pmatrix}10\\10\end{pmatrix} - \begin{pmatrix}-14\\-5\end{pmatrix} = \begin{pmatrix}24\\-10\end{pmatrix}$ ✱

(b) $\underline{V}_{WB} = \underline{V}_W - \underline{V}_B \Rightarrow \underline{V}_W = \underline{V}_{WB} + \underline{V}_B$

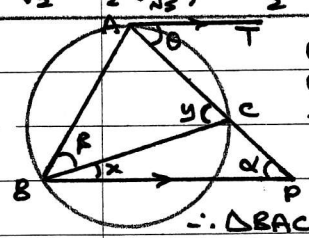
$\therefore \underline{V}_W = \frac{1}{2}V_1\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} + \begin{pmatrix}0\\u\end{pmatrix} = \begin{pmatrix}V_2\\0\end{pmatrix} + \frac{u}{2}\begin{pmatrix}-\sqrt{3}\\1\end{pmatrix}$

$\therefore \frac{1}{2}V_1(-\sqrt{3}) + u = \frac{u}{2} \Rightarrow V_1 = \frac{u}{\sqrt{3}}$

$\therefore \underline{V}_W = \frac{1}{2}\left(\frac{u}{\sqrt{3}}\right)\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} + \begin{pmatrix}0\\u\end{pmatrix} = \frac{u}{2\sqrt{3}}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$ ✱

$V_2 = \frac{1}{2}\left(\frac{u}{\sqrt{3}}\right) + \frac{u\sqrt{3}}{2} = \frac{2\sqrt{3}}{3}u$ ✱

6.



$\theta = \alpha$ (Alternate ∡, AT // BP)

$\theta = \beta$ (∡ in alternate segment)

$\therefore \alpha = \beta$

$y = x + \alpha$ (ext. ∡ of $\triangle BCP$)

$= x + \beta$

$\measuredangle BAC = \measuredangle PAB$ [common angle]

$\therefore \triangle BAC \sim PAB$

$\frac{AB}{AC} = \frac{AP}{AB} \Rightarrow AP \cdot AC = AB^2$ ✱

7.(a) No. of CW $= {}^5P_4 = 120$ ✱

(b) No. of CW $= {}^5C_2\left(\frac{4!}{2!}\right) = 120$ ✱

8.(a) Prob. $= \frac{1}{3}$ ✱

(b) Let X = no. of correct answer Qs, $X \sim B(20, \frac{1}{3})$

$E(X) = 20\left(\frac{1}{3}\right) = \frac{20}{3}$ ✱, $Var(X) = \frac{20}{3}\left(\frac{2}{3}\right) = \frac{40}{9}$ ✱

$X \sim N\left(\frac{20}{3}, \frac{40}{9}\right)$: $P(X \geq k) \leq 0.01$

$P\left(Z \geq \left(k - 0.5 - \frac{20}{3}\right) \Big/ \frac{2}{3}\sqrt{10}\right) \leq 0.01$

$\therefore k - 0.5 - \frac{20}{3} \geq \frac{2}{3}\sqrt{10}\,(2.326)$

$\therefore k \geq 12.07 \Rightarrow$ Passing mark $= 13$ ✱

9. Let X = no. of delay in a week, $X \sim P_o(1)$

$T = X_1 + X_2 + X_3 + X_4$: $T \sim P_o(4)$

$P(T \geq 4) = 1 - e^{-4}\left[1 + 4 + \frac{4^2}{2} + \frac{4^3}{6}\right] = 0.567$ ✱

Y = no. of 4 week with ($\geq$ 4 delays), $Y \sim B(13, 0.567)$

$P(Y \geq 10) = {}^{13}C_{10}(0.567)^{10}(0.433)^3 +$

${}^{13}C_{11}(0.567)^{11}(0.433)^2 + 13(0.567)^{12}(0.433) + (0.567)^{13}$

$= 0.115$ ✱

$U = X_1 + X_2 + \ldots + X_n$, $U \sim P_o(n)$

$P(U \geq 1) > 0.995$: $e^{-n} < 0.005$

$\therefore n > -\ln 0.005 = 5.30 \Rightarrow n_{min} = 6$ ✱

10.(a) $S = X_1 + \ldots + X_{100}$: $S \sim N(100(3), 100(4))$

$P(S > 310) = P\left(Z > \frac{310 - 300}{20}\right) = 0.3085$ ✱

(b) $3S - T \sim N(3(300) - 300(3), 3^2(400) + 300(4))$

$P(3S > 50 + T) = P(3S - T > 50)$

$= P\left(Z > \frac{50 - 0}{\sqrt{4800}}\right) = 0.2352$ ✱

$Q = X_1 + X_2 + \ldots + X_n$: $Q \sim N(n(3), n(4))$

$P(Q > 3.5n) = P\left(Z > \frac{3.5n - 3n}{2\sqrt{n}}\right) = P\left(Z > \frac{\sqrt{n}}{4}\right)$

$\therefore \lim_{n\to\infty} P(Q > 3.5n) = P\left(Z > \lim_{n\to\infty}\frac{\sqrt{n}}{4}\right)$

$= 0$ ✱

11.(a) $P(M=1) = p$, $P(M=2) = (1-p)p$

$P(M=3) = (1-p)^2 p$ ✱

(b) $\therefore P(M=k) = \begin{cases}(1-p)^{k-1}p & k = 1, 2, \ldots \\ 0 & \text{otherwise}\end{cases}$

$\sum P(M=k) = p + (1-p)p + (1-p)^2 p + \ldots$

$= \frac{p}{1-(1-p)} = 1$

since $0 < p < 1$: $(1-p)^{k-1}p > 0 \Rightarrow P(M=k) > 0$

$\therefore M$ is a discrete RV ✱

$P(|M - 4| < 2\sqrt{3}) = P(M = 1, 2, \ldots 7)$

$= \frac{p[1-(1-p)^7]}{1-(1-p)} = 1 - \left(\frac{3}{4}\right)^7 = \frac{14197}{16384}$ ✱

12.(a) $\bar{x} = \frac{66}{30} + 72 = 74.2$ ✱

(b) mean $= \frac{74.2(30) + [90(80) - 135]}{120}$

$= 77.425$ ✱

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