MALACCA (2005)
MATHEMATICS T (PAPER 1)

1. Find the exact value of $\int_0^{\frac{\pi}{2}} \sin x\sqrt{\cos x}\,dx$. [4]

2. Obtain the first three terms in the expansion, in ascending powers of x, of $(8 + 3x)^{\frac{2}{3}}$, stating the set of values of x for which he expansion is valid. [5]

3. Given that $z_1 = 2 + i$, $z_2 = 2 - i$ and $z_3 = 1 + 2i$. Find the modulus and argument of each of the following complex numbers.
   (a) $z_1^2 z_2^2$ (b) $\frac{z_3}{z_1}$ [6]

4. Given that $P = \begin{pmatrix} 1 & 1 & 1 \\ a & 2 & 3 \\ a^2 & 4 & 9 \end{pmatrix}$. Find all the possible values of a such hat P does not have an inverse. [5]

5. The function f is defined by $f(x) = \begin{cases} x-2 & , x \le 2 \\ \frac{1}{x-1} & , x > 2 \end{cases}$.
   Find $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$.
   Hence, determine whether f is continuous at $x = 2$.
   Sketch the graph of f.]6]

6. The points A, B have coordinates $(a, 0)$ and $(-a, 0)$ respectively. A point P in the plane of the coordinates axes moves such that $AP : PB = \lambda : 1$ where λ is a positive constant. Find the equation of the locus of P.
   If $\lambda \neq 1$, show that this locus is a circle, and also show that the centre of this circle lies on the x–axis, but not between A and B. [7]

7. Solve the equation $4|x| = |x - 1|$. [3]
   On the same diagram, sketch the graphs of $y = 4|x|$ and $y = |x - 1|$.
   Hence, solve the inequality $4|x| > |x - 1|$. [4]

8. On the same diagram, sketch the curves with equations $y^2 = 9x$ and $y = x^2(1 + x)$.
   Deduce that the equation $x^3(1 + x) - 9 = 0$ has exactly one real root. [3]
   Denoting this root by α, find an integer n such that $n < \alpha < n + 1$, and taking n as a first approximation, use the Newton-Raphson method to find this root correct to 3 decimal places. [6]

9. If $\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, find $\mathbf{A}^2$. [2]
   Find the values of m and n such that $\mathbf{A}^2 + m\mathbf{A} + n\mathbf{I} = \mathbf{0}$, where $\mathbf{I}$ is the 3 x 3 identity matrix. [5]
   Hence, find $\mathbf{A}^3$ and $\mathbf{A}^{-1}$. [4]

10. A curve has parametric equations $x = 2t$ and $y = \frac{1}{t^2}$, where $t \neq 0$. Sketch the curve.
    Find the equations of the tangent and normal to the curve at the point $(2t, \frac{1}{t^2})$.
    The tangent at the point $P(4, \frac{1}{4})$ meets the y–axis at T, and the normal at P meets the y–axis at N. Find the area of the triangle PTN. [4]

11. The region R, in the first quadrant, is bounded by the line $y = 3 - x$ and the curve $y = \frac{3}{1+x}$ between their points of intersection.
    Find the coordinates of these points, and sketch the region of R. [5]
    Find the volume of the solid formed when R is rotated through one revolution about the x–axis. [4]

12. (a) Find the sum of the arithmetic progression 1, 4, 7, 10, 13, 16, . . . , 1000.
    If every third terms of the above progression are removed (i.e. 7, 16, . . . etc), find the sum of the remaining terms. [6]
    (b) The rth term, U_r, of a series is given by $U_r = \left(\frac{1}{3}\right)^{3r-2} + \left(\frac{1}{3}\right)^{3r-1}$.
    Express $\sum_{r=1}^{n} U_r$ in the form $A(1 - \frac{B}{27^n})$, where A and B are constants.
    Find the sum to infinity of the series. [9]

MATHEMATICS T (PAPER 2)

1. Express $2\sin^2 x + \sin 2x$ in the form $A - R\cos(2x + \alpha)$, where $R > 0$, $0 < \alpha < \frac{\pi}{2}$, and A is a constant.
   Hence, find the set of values of x which satisfy the inequality
   $2\sin^2 x + \sin 2x < 2$, for $0 \le x \le 2\pi$. [7]

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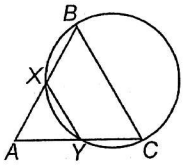
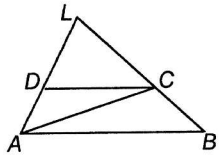
2. In triangle ABC, $AB = BC$ and $AC = 30$ cm. The circle with diameter BC intersects AB at X and AC at Y. Determine the length of XY. [5]



3. A, B and C are three points on horizontal ground. B is 60 m north of A, and C is 25 m west of B. PB and QC are two vertical poles of height 10 m and 15 m respectively. D is due east of B in such a way that the angle of elevation of Q and the angle of elevation of P from D are equal. Find
   (i) the angle of elevation of Q from D. [2]
   (ii) the area of triangle ACD. [3]
   (iii) the bearing of D from A. [3]
   (iv) the angle between the plane AQD and the horizontal plane. [3]

4. At 0900, a boat A is 10 km east of another boat B. Boat A is sailing with speed 45 kmh^{-1} in the direction $N\ 60^\circ\ E$. If boat B is sailing with speed 60 kmh^{-1},
   (i) determine the direction of B if the boats will collide in subsequent motion. [4]
   (ii) determine also the time when the boats collide. [4]

5. In trapezium $ABCD$, the sides AB and DC are parallel, and $\measuredangle DAC = \measuredangle ABC$.
   (i) Prove that ΔACD and ΔBAC are similar. [3]
   (ii) If it is further given that AD and BC are produced to meet at L and $DL = 2\,AD$, show that $\frac{DC}{AB} = \frac{2}{3}$.
   Hence, prove that $AC^2 = \frac{2}{3}AB^2$. [7]



6. Using the substitution $y = vx$, solve the differential equation $x\frac{dy}{dx} = y + x^2$, for $y = 0$ when $x = -1$, expressing y in terms of x. [6]

7. At each turn in a game, a player's score, denoted by the random variable X, is found as follows : The player throws a fair die once.
   If the number shown on the die is 5 or 6, then that number is the value of X.
   If the number shown on the die is 1, 2, 3 or 4, then the player throws the die again, and X is the number shown on the second throw.
   (i) Show that $P(X = 1) = \frac{1}{9}$ and $P(X = 6) = \frac{5}{18}$. [3]
   (ii) Tabulate the probability distribution of X. [2]
   If two players P and Q take turns to play the game, find the probability that the two players obtain the same scores after a game played. [3]

8. A manufacturer of plastic plates finds that on average one plate in every fifty is fault. If two hundred plastic plates are taken randomly from the factory, using an appropriate approximation, calculate the probability that not more than two of the plates are faulty. [4]

9. A company employs three grades of manual workers. The type of employee and the percentage of total manual workers are as follows.
   Skilled 20%, semi-skilled 30% and unskilled 50%. Company records show that 80% of the skilled manual workers had at least one qualification, the corresponding percentage for semi-skilled and unskilled being 50% and 20% respectively.
   (i) Calculate the percentage of the manual work force that had at least one qualification. [2]
   (ii) A manual worker is selected at random from the work force. If the person selected has at least one qualification, what is the probability that the person is a skilled manual worker? [3]

10. A machine produces sheets of glass of thickness X mm, where $X \sim N(\mu, \sigma^2)$. Over a long period of time it has been found that 2% of the sheets are less than 2 mm thick and 5% of the sheets are more than 3 mm thick. Find, correct to 2 decimal places, the values of μ and σ. [6]
    In the manufacture of a certain type of windscreen, two of these sheets are taken at random and put together to form a sheet of double thickness.
    (i) Find, correct to 2 decimal places, the probability that the thickness of a sheet of double thickness will lie between 4 mm and 6 mm. [4]
    (ii) Given that a sheet of double thickness is between 4 mm and 6 mm thick. Find, correct to 2 decimal places, the probability that the two single sheets from which it is made are each between 2 mm and 3 mm thick. [3]

11. The table shows the time spent (in minutes) to complete a construction puzzle by a random sample of 95 participants from various occupations organized by a research group.

Time taken	10 –	20 –	30 –	40 –	50 –	60 –	70 –	80 – 100
No. of participants	5	11	16	33	12	9	6	3

    (a) Draw a cumulative frequency diagram to represent these data. [3]
    (b) Calculate the values of mean and standard deviation. [5]
    It is decided to grade the participants based on their times taken as good, average and poor. A method states that the percentages of participants in these grades are to be approximately 15%, 70% and 15% respectively. Estimate the grade limits. [3]

12. The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} k & ,2 \le x < 3 \\ k(x-2) & ,3 \le x < 4 \\ 0 & ,\text{otherwise} \end{cases}$$

, where k is a constant.
    (i) Determine the value of k. [3]
    (ii) Find the cumulative distribution function of X, $F(x)$, hence determine the median. [7]
    (iii) Find the probability that three independent observations of X has exactly one value greater than median. [2]

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Malacca (2005)

MT1

No. Date:

1. Let $u = \cos x$: $\frac{du}{dx} = -\sin x$

$\int_0^{\pi/2} \sin x \sqrt{\cos x}\, dx = \int_1^0 -\sqrt{u}\, du$

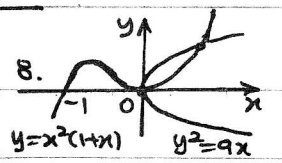
$= -\frac{2}{3}\left[u\sqrt{u}\right]_1^0 = -\frac{2}{3}[0-1] = \frac{2}{3}$ ✱

2. $(8+3x)^{2/3} = 4\left(1+\frac{3}{8}x\right)^{2/3}$

$= 4\left[1 + \frac{2}{3}\left(\frac{3}{8}x\right) + \frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)}{2!}\left(\frac{3}{8}x\right)^2 + \dots\right]$

$= 4 + x - \frac{1}{16}x^2 + \dots$ ✱

8.



$x^3(1+x)^2 - 9 = 0$ — ①

$\therefore [x^2(1+x)]^2 = 9x$

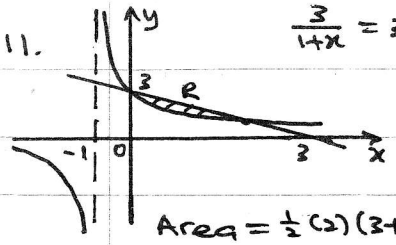
$\therefore$ $y = x^2(1+x)$ and $y^2 = 9x$ has 2 intersection points, but $x=0$ is not a root for ①

$\therefore$ ① has only 1 real root. ✱

Let $f(x) = x^3(1+x)^2 - 9$:

$f(1) \cdot f(2) = (-5)(63) < 0 \Rightarrow 1 < \alpha < 2$

11.



$\frac{3}{1+x} = 3-x \Rightarrow x^2-2x=0$

$\therefore x=0$ or 2

Intersection points $(0,3)$ and $(2,1)$ #

$\text{Area} = \frac{1}{2}(2)(3+1) - \int_0^2 \frac{3}{1+x}\,dx$

$= 4 - 3[\ln(1+x)]_0^2 = 4 - 3\ln 3$ #

$V_x = \pi\int_0^2 \left[(3-x)^2 - \frac{9}{(1+x)^2}\right]dx$

$= \pi\left[-\frac{(3-x)^3}{3} + \frac{9}{1+x}\right]_0^2$

$= \pi\left[-\frac{1}{3}+9+3-9\right] = \frac{8}{3}\pi$ #

12.(a) $\text{Sum} = [1+4+7+\dots+1000]$

$-[7+16+25+\dots+997]$

$= 3\left[\frac{1}{3}+1\frac{1}{3}+2\frac{1}{3}+\dots+333\frac{1}{3}\right] - 9\left[\frac{7}{9}+1\frac{7}{9}+\dots+110\frac{7}{9}\right]$

$= 3\left(\frac{334}{2}\right)\left(\frac{1}{3}+333\frac{1}{3}\right) - 9\left(\frac{111}{2}\right)\left(\frac{7}{9}+110\frac{7}{9}\right)$

$= 111445$ #

(b) $u_r = \left(\frac{1}{3}\right)^{3r-2} + \left(\frac{1}{3}\right)^{3r-1} = \left(\frac{1}{3}\right)^{3r}[9+3]$

$= \left(\frac{1}{3}\right)^{3r}[12]$

$\therefore \sum_{r=1}^{n} u_r = 12\left[\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^6 + \dots + \left(\frac{1}{3}\right)^{3n}\right]$

$= 12\left[\frac{\frac{1}{27}\left(1-\left(\frac{1}{27}\right)^n\right)}{1-\frac{1}{27}}\right] = \frac{6}{13}\left[1-\frac{1}{27^n}\right]$ #

$\sum_{r=1}^{\infty} u_r = \lim_{n\to\infty}\left[\frac{6}{13}\left(1-\frac{1}{27^n}\right)\right]$

$= \frac{6}{13}[1-0] = \frac{6}{13}$ #

1. $f(x) = 2\sin^2 x + \sin 2x = 1 - [\cos 2x - \sin 2x]$

$g(x) = \cos 2x - \sin 2x = R\cos(2x+\alpha)$

$= R\cos 2x\cos\alpha - R\sin 2x\sin\alpha$

$\therefore R\cos\alpha = 1,\ R\sin\alpha = 1$

$\therefore \tan\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$ $\quad\therefore R = \sqrt{2}$

$\therefore f(x) = 1 - \sqrt{2}\cos\left(2x+\frac{\pi}{4}\right)$ #

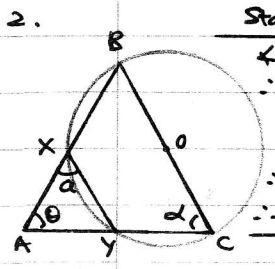
For $f(x) < 2$: $1-\sqrt{2}\cos\left(2x+\frac{\pi}{4}\right) < 2$

$\therefore \cos\left(2x+\frac{\pi}{4}\right) > -\frac{1}{\sqrt{2}}$

$\therefore \frac{\pi}{4} \le 2x+\frac{\pi}{4} < \frac{3\pi}{4}$ or $\frac{5\pi}{4} < 2x+\frac{\pi}{4} \le \frac{9\pi}{4}$

$\therefore \{x : 0 \le x < \frac{\pi}{4}$ or $\frac{\pi}{2} < x \le \pi\}$ #

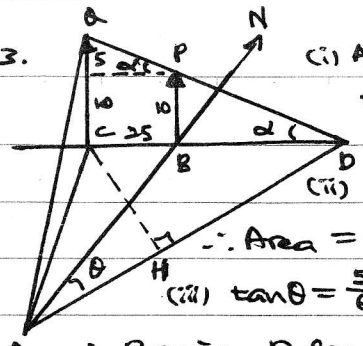
2.



Statements	Reasons
$\angle BYC = 90°$	Angle in semicircle
$\therefore BYA = 90°$	supplementary angle.
$AB = CB$	given
$\theta = \alpha$	ΔABC is a scales
$\therefore \Delta BYA \equiv \Delta BYC$	(RHS Theorem)
$\therefore AY = YC = 15$	
$a = \alpha$	exterior angle of cyclic BCYX.

$\therefore \Delta AXY$ is isosceles with $XY = AY = 15$cm #

3.



(i) Angle of elevation θ from D $= \tan^{-1}\left(\frac{5}{25}\right) = 11.3°$ #

(ii) $BD = 10\cot 11.3° = 50$

$\therefore \text{Area} = \frac{1}{2}(60)(75) = 2250\,m^2$ #

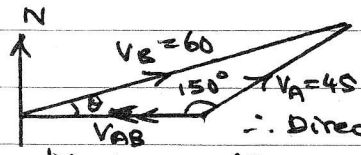
(iii) $\tan\theta = \frac{50}{60} \Rightarrow \theta = 39.8°$

$\therefore$ Bearing D from A $= N\,39.8°\,E$ #

(iv) Angle $= \tan^{-1}\left(\frac{15}{CH}\right) = \tan^{-1}\left[\frac{15}{75\sin 50.2°}\right]$

$= 14.6°$ #

4(i) To Collide: V_{AB} to west



$\frac{\sin\theta}{45} = \frac{\sin 150°}{60}$

$\therefore \theta = 22.0°$

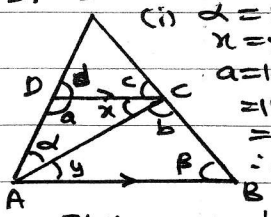
$\therefore$ Direction of B $= N\,68.0°\,E$ #

(ii) $\frac{|V_{AB}|}{\sin 8.0°} = \frac{60}{\sin 150°} \Rightarrow |V_{AB}| = 16.70\,kmh^{-1}$

$\therefore$ time taken $= \frac{10}{16.70}\,h = 36$ min.

$\therefore$ time $= 0936$ #

5.



Statements	Reasons
(i) $\alpha = \beta$	given
$x = y$	alternate angles
$a = 180° - x - \alpha$	sum of angles in ΔACD
$= 180° - y - \beta$	
$= b$	sum of angle in ΔACB
$\therefore \Delta ACD \sim \Delta BAC$	$\left[\Rightarrow \frac{AC}{AB} = \frac{CD}{AC}\right]$

#

Statements	Reasons
$d = \alpha + y$	Corresponding angles
$c = \beta$	Corresponding angles
$\angle L = \angle L$	Common angle

$\therefore \Delta ALB \sim \Delta DLC \Rightarrow \frac{DC}{AB} = \frac{DL}{AL} = \frac{2AD}{3AD} = \frac{2}{3}$ #

$\therefore AC^2 = AB.CD = AB.\frac{2}{3}AB = \frac{2}{3}AB^2$ #

6. $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore x\frac{dy}{dx} = x\left(v + x\frac{dv}{dx}\right) = vx + x^2 \Rightarrow \frac{dv}{dx} = 1$

$\therefore \int_0^v dv = \int_{-1}^{x} dx \Rightarrow [v]_0^v = [x]_{-1}^{x}$

$\therefore v = x+1 = \frac{y}{x} \Rightarrow y = x^2 + x$ #

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7. (i) $P(X=1) = (\frac{4}{6})(\frac{1}{6}) = \frac{1}{9}$

$P(X=6) = \frac{1}{6} + (\frac{4}{6})(\frac{1}{6}) = \frac{5}{18}$

(ii)

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{5}{18}$

(iii) Prob. $= (\frac{1}{9})^2(4) + (\frac{5}{18})^2(2) = \frac{11}{54}$

8. X = no of faulty, $X \sim B(200, \frac{1}{50})$

$n = 200 (>50)$, $np = 4 (<5)$: $X \sim Po(4)$

$P(X \le 2) = e^{-4}[1 + 4 + \frac{4^2}{2!}] = 0.2381$

9. (i) $P(Q) = 0.2(0.8) + 0.3(0.5) + 0.5(0.2)$

$= 0.41$

$\therefore$ Percentage $= 41\%$

(ii) $P(S|Q) = \frac{0.2(0.8)}{0.41} = \frac{16}{41}$

10. $X \sim N(\mu, \sigma^2)$

$P(X<2) = 0.02$: $\frac{2-\mu}{\sigma} = -z_{0.02} = -2.054$

$\therefore 2 - \mu = -2.054\sigma$ — ①

$P(X>3) = 0.05$: $\frac{3-\mu}{\sigma} = z_{0.05} = 1.645$

$\therefore 3 - \mu = 1.645\sigma$ — ②

② − ①: $1 = 3.699\sigma \Rightarrow \sigma = 0.27$

$\therefore \mu = 2 + 2.054(\frac{1}{3.699}) = 2.56$

$T = X_1 + X_2 \sim N(2\mu, 2\sigma^2)$

(i) $P(4<T<6) = P(\frac{4-5.12}{0.27\sqrt{2}} < Z < \frac{6-5.12}{0.27\sqrt{2}})$

$= 0.9983 - 0.0106 = 0.99$ (2dp)

(ii) $P(2<X<3) = P(\frac{2-2.56}{0.27} < Z < \frac{3-2.56}{0.27})$

$= 0.9810 - 0.0516 = 0.93$ (2dp)

$P(2<2 \text{ & } 3 \text{ meet each} < 3 \mid 4<T<6)$

$= \frac{(0.93)^2}{0.99} = 0.87$ (2dp)

11. (b) $n = 95$, $\Sigma fx = 4360$

$\Sigma fx^2 = 226800$

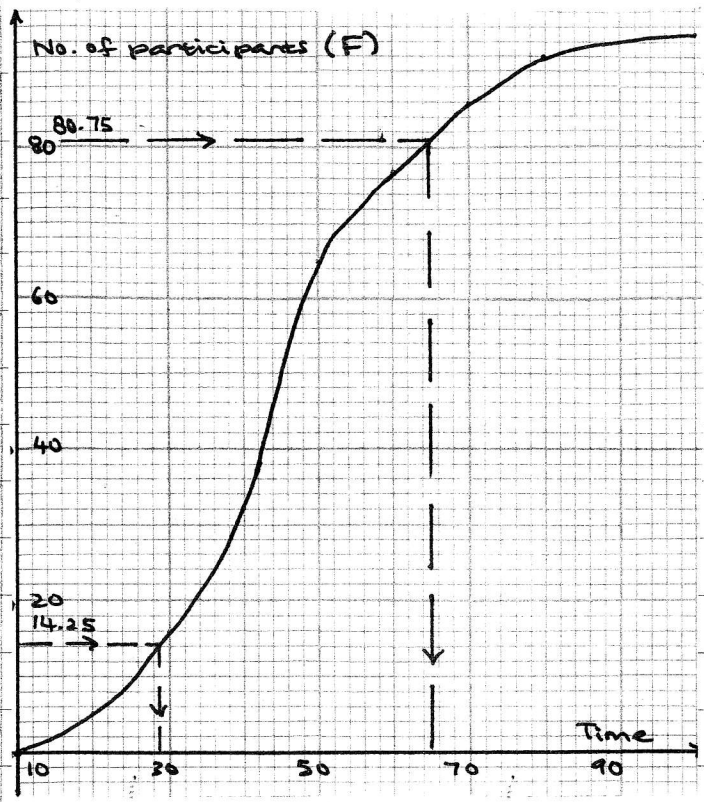
$\therefore \bar{x} = \frac{4360}{95} = 45.89$

$\therefore s = \sqrt{\frac{226800}{95} - (\frac{4360}{95})^2} = 16.76$

Good : $10 \le x < 28$

Average : $28 \le x < 65$

Poor : $65 \le x \le 100$



12. (i) $\int_2^3 k\,dx + \int_3^4 k(x-2)\,dx = 1$

$\therefore k[x]_2^3 + k[\frac{(x-2)^2}{2}]_3^4 = 1$

$\therefore k(3-2) + k[2 - \frac{1}{2}] = 1 \Rightarrow k = \frac{2}{5}$

(ii) $2 \le x < 3$: $P(X \le x) = \int_2^x \frac{2}{5}\,dx = \frac{2}{5}(x-2)$

$3 \le x < 4$: $P(X \le x) = \frac{2}{5}(3-2) + \int_3^x \frac{2}{5}(x-2)^2\,dx$

$= \frac{2}{5} + \frac{1}{5}[(x-2)^2]_3^x = \frac{1}{5}[(x-2)^2 + 1]$

$$\therefore F(x) = \begin{cases} 0 & x < 2 \\ \frac{2}{5}(x-2) & 2 \le x < 3 \\ \frac{1}{5}[(x-2)^2+1] & 3 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

$P(X<m) = \frac{1}{2}$: $(3 \le m < 4)$

$\therefore \frac{1}{5}[(m-2)^2 + 1] = \frac{1}{2} \Rightarrow (m-2)^2 = \frac{3}{2}$

$\therefore m = 2 + \frac{\sqrt{3}}{2}$ $[\because 3 \le m < 4]$

(iii) Prob. $= (\frac{1}{2})(\frac{1}{2})^2(\frac{3!}{2!}) = \frac{3}{8}$

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