LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK JIT SIN (2005)
MATHEMATICS T (PAPER 1)

1. Given that $y = \log_2 x$ and $\log_2 x - \log_x 8 + \log_2 2^k + k\log_x 4 = 0$.
Prove that $y^2 + ky + (2k-3) = 0$.

Hence, deduce the set of values of k for which y is real.

Find the value of x when $k = \frac{3}{2}$. [8]

2. The complex numbers p and q are given by $p = 3 - i$ and $q = 1 - 2i$.

Find the modulus and argument of $\frac{p}{q}$. Hence write down the value of $\left(\frac{p}{q}\right)^{20}$.

Sketch in an Argand diagram, the locus of the point Z representing the complex number z such that $|z - p| = |q - p|$, where p and q are the fixed complex numbers given above. [8]

3. Find the set of values of x for which

(a) $\frac{|x|}{|x|-2} < 5$ [4] (b) $\left|\frac{x}{5+x}\right| < 3$ [3]

4. The roots of the equation $x^2 + ax + b = 0$ are α and β.
Find both the roots of the equation $4bx^2 + 2ax + 1 = 0$ in terms of α and β. [5]

5. An athlete is considering two plans for a training programme.
Plan I : Run 2000 m on day 1, 2400 m on day 2, and on each successive day increase the distance run by 400 m.
Plan II : Run 2000 m on day 1, 2200 m on day 2, and on each successive day multiply the distance run by a constant factor $\frac{11}{10}$.

(a) For Plan I, find in the form $A(n + B)$, where A and B are constants to be determined, the distance, in metres, run on day n. [2]

(b) For Plan I, find in the form $Cn(n + D)$, where C and D are constants to be determined, the total distance, in metres, run on days 1 to n, inclusively. [2]

(c) For Plan I, find the least value of n for which the total distance run on days 1 to n, inclusively, exceeds 200 000 m. [3]

(d) For Plan II, find an expression for the distance, in metres, run on day N. [1]

(e) For Plan II, find the least value of N for which the distance run on day N exceeds 16 000 m. [4]

6. Express $f(x) = \frac{7 - 3x - x^2}{(1-x)^2(2+x)}$ in partial fraction.

Hence, prove that, if x^3 and higher powers of x may be neglected, then

$f(x) = \frac{1}{8}(28 + 30x + 41x^2)$. [7]

7. (a) If $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$, show that $\mathbf{A}^3 = 3\mathbf{A}$. Hence find $\mathbf{A}^{10}$. [4]

(b) Given that $\mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ s & 0 & t \\ k & 1 & 1 \end{pmatrix}$ is a symmetric matrix, find the values of s, k and t. [3]

(c) For matrix $\mathbf{C} = \begin{pmatrix} -2 & 2 & 4 \\ 0 & -2 & -1 \\ 3 & 1 & 3 \end{pmatrix}$, show that $\mathbf{C}$ is not singular. Find *adj* $\mathbf{C}$ and $\mathbf{C}^{-1}$. [5]

8. (a) The straight line L_1 passes through the point P(–2, 0) and Q(–1, –3), and intersects the y–axis at a point R. The straight line L_2 passes through a point Q and is perpendicular to the line L_1. Given that L_2 intersects the x–axis and the y–axis at the point M and N respectively, show that RN : MN = $\sqrt{10}$: 8. [8]

(b) Find the equation of the tangent to the rectangular hyperbola $xy = c^2$ at the point $P\left(cp, \frac{c}{p}\right)$. By assuming $p > 1$, find in terms of p and c, the perpendicular distance of the origin O, from the tangent. [5]

9. The functions f and g are defined as follows :
$f : x \longrightarrow x^2 + 2x,\ x \geq 1$ and $g : x \longrightarrow x + 4,\ x \in \mathcal{R}$.

(a) Find the value of x such that $gf(x) = 7$. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

10. The function f is defined by $f : x \longrightarrow \begin{cases} 2|x-4| & , x < 2 \\ 4x - x^2 & , x \geq 2 \end{cases}$.

(a) Without using graph, determine whether f is continuous at $x = 2$. [4]

(b) Sketch the graph of f. [3]

11. Given that $y = e^{-2x}\sin 3x$. Show that $\frac{dy}{dx} = 3e^{-2x}\cos 3x - 2y$, and hence show that $\frac{d^2y}{dx^2} = 13y - 4\frac{dy}{dx}$. [5]

12. Find the equations of the asymptotes of the graph $y = \frac{3-2x}{x-2}$ and sketch the graph. [4]

On the same diagram, sketch the graph of $y = 1 - e^{-2x}$. [1]

Show that, where the graphs intersect, $(3x-5)e^{2x} = x - 2$, and hence state the number of real roots of this equation. [2]

By taking $x = 1.7$ as a first approximation, use the Newton–Raphson method once to obtain a second approximation to one of the roots of the equation. Give your answer in three significant figures. [4]

LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

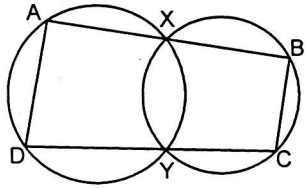
SMJK JIT SIN (2005)
MATHEMATICS T (PAPER 2)

1. Solve the equation $3\cos 2x - \sin x - 2 = 0$ for $0 \le x \le 2\pi$, and give your answer correct to the nearest 0.1 radian. [5]

2. Given that $\tan\theta + \cot\theta = \frac{5}{2}$, find the value of $\tan^2\theta + \sec\theta\,\mathrm{cosec}\,\theta + \cot^2\theta$ in its simplest fraction form. [6]

3. Solve the differential equation $\frac{dy}{dx} = y^2e^x$, given that $y = 1$ when $x = 0$. Give your answer by expressing y in terms of x. [5]

4. Two vectors are given by $\underline{x} = 2\underline{i} + 3\underline{j}$ and $\underline{y} = k\underline{i} - 4\underline{j}$. Determine the values of k such that
   (a) $\underline{x}$ and $\underline{y}$ are perpendicular to each other. [2]
   (b) the acute angle between $\underline{x}$ and $\underline{y}$ is 60°, leave your answer in surd form. [3]

5. 

In the given diagram, XY is the common chord of two intersecting circles. AXB and DYC are straight lines. Prove that AD is parallel to BC. [6]

6. A warship is travelling north west at a constant speed of 5 ms^{-1}. A cannon mounted on the ship can fire a shot with horizontal speed of 25 ms^{-1} relative to the ship. It is required to hit a stationary target which, at the time of firing, is due east of the ship. Find the direction in which the cannon should be aimed. (The vertical motion of the shot should be ignored.) [5]

   The stationary target now moves southwards at a constant speed of 3 ms^{-1}. The warship changes its course in order to intercept the target. Determine its new direction. [5]

7. A bag contains 3 brown balls, 4 yellow balls and 2 pink balls. Four balls are drawn at random without replacement from the bag. Find
   (i) the probability that the balls drawn are all of the same colour. [2]
   (ii) the probability that at least one ball of each colour is drawn. [3]

8. A random variable X has cumulative distribution function F(x) where

$$F(x) = \begin{cases} 0 & , \quad x < -1 \\ kx + k & , -1 \le x < 0 \\ 2kx + k & , \ 0 \le x < 1 \\ 3k & , \ 1 \le x \end{cases}$$

   Determine
   (i) the value of k. [1]
   (ii) the probability density function $f(x)$ of X. [2]
   (iii) the expected value μ of X. [3]
   (iv) the standard deviation σ of X. [3]
   (v) the probability that $|X - \mu|$ exceeds $\frac{1}{3}$. [3]

9. The probability that a person who will be infected by a certain disease is 0.1. A random sample of 5 people are taken. Find the probability that none of them is infected by the disease. [3]

   If 50 people are taken, use a suitable approximation to estimate the probability that less than 4 people will be infected. [3]

   A random sample of n people are now investigated. The probability that at most $(n - 1)$ people are infected was found to be more than 0.99985. Determine the smallest value of n. [4]

10. A firm manufactures both short and long bricks. The length of the short bricks has a normal distribution with mean 20 cm and standard deviation 1.2 cm. The length of the long bricks has a normal distribution with mean k cm and standard deviation 1.8 cm. The random variable X is the combined length of a random sample of 8 short bricks, and the random variable Y is the combined length of a random sample of 4 long bricks.
    (a) Determine P(X > 170). [3]
    (b) Find the mean and variance of X – Y. [2]
    (c) Given that P(X > Y) = 0.9, find the value of k, correct to the nearest integer. [3]
    (d) By using the value of k found in (c), determine P(2X > Y + 150). [4]

11. A discrete random variable X can take only the values 0, 1, 2 or 3, and its probability distribution is given by P(X = 0) = 2k, P(X = 1) = k, P(X = 2) = 3k, P(X = 3) = 4k, where k is a constant. Find
    (a) the value of k. [1]
    (b) the mean and variance of X [5]
    (c) the values of E(2X – 1) and Var(2X – 1). [2]
    Determine the cumulative distribution function of X. [2]

12. The following table shows the number of cars sold by agents of a car company during a period of one week.

No. of cars	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30
No. of agents	1	6	9	18	12	4

    (a) Determine the mean and variance of the number of cars sold in one week. [4]
    (b) Calculate its median and semi-interquartile range. [8]
    (c) Any agent who sells at least 21 cars in one week will be awarded a special prize. Calculate the percentage of the agents who won the prize. [2]

SMJK Jit Sin (2005)
MT/MS1

No: ..... Date: .....

1. $y=\log_2 x$: $\log_2 x - \log_x 8 + \log_2 2^k + k\log_x 4 = 0$

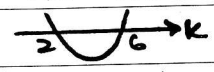
$\therefore y - \frac{3}{y} + k + k\left(\frac{2}{y}\right) = 0$ —(×y)

$\therefore y^2 + ky + (2k-3) = 0$ ✱

For y real: $b^2-4ac \geq 0 \Rightarrow k^2-4(2k-3)\geq 0$

$k^2-8k+12\geq 0 \Rightarrow (k-2)(k-6)\geq 0$

$\therefore \{k : k\leq 2 \text{ or } 6\leq k\}$ ✱



$k=\frac{3}{2}$: $y^2+\frac{3}{2}y=0 \Rightarrow y=0$ or $-\frac{3}{2}$

$\therefore x = 2^0$ or $2^{-3/2}$

$[\because x\neq 1]$: $x=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}$ ✱

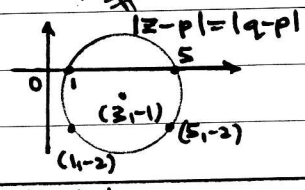
2. Let $w=\frac{p}{q}=\frac{3-i}{1-2i}\cdot\frac{1+2i}{1+2i}=1+i$

$\therefore |w|=\sqrt{2}$ ✱, $\arg(w)=\frac{\pi}{4}$ ✱

$w^2=2i$, $w^4=-4$

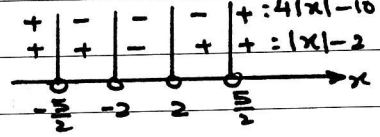
$\therefore w^{20}=(-4)^5=-1024$ ✱

$|q-p|=|-2-i|=\sqrt{5}$



3(a) $\frac{|x|}{|x|-2}<5 \Rightarrow \frac{5|x|-10-|x|}{|x|-2}>0$

$\therefore \frac{4|x|-10}{|x|-2}>0$



$x\in(-\infty,-\frac{5}{2})\cup(-2,2)$
$\cup(\frac{5}{2},\infty)$ ✱

(b) $\left|\frac{x}{5+x}\right|<3 \Rightarrow |x|<3|5+x|$ $(x\neq -5)$

$\therefore x^2<9(25+10x+x^2)$

$\therefore 8x^2+90x+225>0$

$\therefore (4x+15)(2x+15)>0$

$\therefore \{x : x<-\frac{15}{2} \text{ or } -\frac{15}{4}<x\}$ ✱

4. $x^2+ax+b=0$ (α,β are roots)

$\therefore \alpha+\beta=-a$, $\alpha\beta=b$

$4bx^2+2ax+1=0$

$\therefore 4\alpha\beta x^2-2(\alpha+\beta)x+1=0$

$\therefore (2\alpha x-1)(2\beta x-1)=0$

$\therefore$ roots $=\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$ ✱

5.(a) $T_n=2000+(n-1)(400)=400(n+4)$ ✱

(b) $S_n=\frac{n}{2}[2000+1600+400n]$

$=200n[n+9]$ ✱

(c) $S_n>200000$: $n(n+9)>1000$

$\therefore (n+\frac{9}{2})^2>1000+\frac{81}{4}$

$[\because n\in\mathbb{N}, n+\frac{9}{2}>0]$: $n+\frac{9}{2}>\sqrt{1020.25}$

$\therefore n>27.44 \Rightarrow n_{min}=28$ ✱

(d) $U_N=2000\left(\frac{11}{10}\right)^{N-1}$ ✱

(e) $U_N>16000$: $1.1^{N-1}>8 \Rightarrow (N-1)\lg 1.1>\lg 8$

$\therefore N>22.82 \Rightarrow N_{min}=23$ ✱

6. Let $f(x)=\frac{7-3x-x^2}{(1-x)^2(2+x)}$

$=\frac{A}{2+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}=\frac{A(1-x)^2+B(1-x)(2+x)+C(2+x)}{(1-x)^2(2+x)}$

$\therefore 7-3x-x^2=A(1-x)^2+B(1-x)(2+x)+C(2+x)$

$x=1$: $7-3-1=C(3) \Rightarrow C=1$

$x=-2$: $7+6-4=A(9) \Rightarrow A=1$

$[x^2]$: $-1=A-B \Rightarrow B=2$

$\therefore f(x)=\frac{1}{2+x}+\frac{2}{1-x}+\frac{1}{(1-x)^2}$ ✱

$\therefore f(x)=\frac{1}{2}(1+\frac{x}{2})^{-1}+2(1-x)^{-1}+(1-x)^{-2}$

$=\frac{1}{2}[1-\frac{x}{2}+(\frac{x}{2})^2]+2(1+x+x^2)+(1+2x+3x^2)$

$=\frac{1}{8}[28+30x+41x^2]$ ✱

7.(a) $A^3=\begin{pmatrix}2&1\\-1&-2\end{pmatrix}\begin{pmatrix}2&1\\-1&-2\end{pmatrix}\begin{pmatrix}2&1\\-1&-2\end{pmatrix}=\begin{pmatrix}2&1\\-1&-2\end{pmatrix}\begin{pmatrix}3&0\\0&3\end{pmatrix}$

$=3AI=3A$ ✱

$A^{10}=(A^3)^3A=27A^3A=81A^2=\begin{pmatrix}243&0\\0&243\end{pmatrix}$ ✱

(b) $B=B^T$: $s=2, k=3, t=1$ ✱

(c) $(C_{ij})=\begin{bmatrix}-5&-3&6\\-3&-18&8\\6&-2&4\end{bmatrix}$

$\therefore |C|=-2(-5)+0+3(6)=28\ (\neq 0) \Rightarrow$ C is not singular. ✱

$\therefore \text{adj } C=(C_{ij})^T=\begin{bmatrix}-5&-2&6\\-3&-18&-2\\6&8&4\end{bmatrix}$ ✱

$C^{-1}=\frac{1}{28}(\text{adj } C)=\frac{1}{28}\begin{bmatrix}-5&-2&6\\-3&-18&-2\\6&8&4\end{bmatrix}$ ✱

8(a) PQ: $\frac{y-0}{x+2}=\frac{0+3}{-2+1} \Rightarrow y=-3(x+2)$ $\therefore R(0,-6)$

L_2: $3y-x=3(-3)-(-1) \Rightarrow 3y-x=-8$

$\therefore M(8,0)$ and $N(0,-\frac{8}{3})$

$RN:MN=6-\frac{8}{3}:\sqrt{8^2+(\frac{8}{3})^2}=\frac{10}{3}:\frac{8}{3}\sqrt{10}$

$=10:8\sqrt{10}=\sqrt{10}:8$ ✱

(b) $xy=c^2 \Rightarrow \frac{dy}{dx}=-\frac{c^2}{x^2}$

$P(cp,\frac{c}{p})$: $m_t=-\frac{1}{p^2}$

Tangent: $y-\frac{c}{p}=-\frac{1}{p^2}(x-cp) \Rightarrow p^2y+x=2cp$ ✱

$d_0=\frac{|2cp|}{\sqrt{p^4+1}}=\frac{2p|c|}{\sqrt{p^4+1}}$ ✱

LEE HOCK LEONG
B. Sc (Ed.) - 1st. Class Hons.
M. Sc (Pure Maths.) - CGPA4

Uni

9. (a) $gf(x)=7$: $f(x)+4=7 \Rightarrow x^2+2x-3=0$

$\therefore (x-1)(x+3)=0 \Rightarrow x=1$ or -3

$[\because x \geq -1]$: $x=1$ ✱

(b) Let $f^{-1}(x)=y$: $x=f(y)=y^2+2y$

$\therefore (y+1)^2=x+1 \Rightarrow y=-1\pm\sqrt{x+1}$

$[\because y \geq -1]$: $f^{-1}(x)=-1+\sqrt{x+1}$ ✱

10. (a) $f(2)=4(2)-4=4$

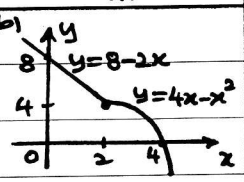
$\lim_{x\to 2^+} f(x)=4(2)-4=4$

$\lim_{x\to 2^-} f(x)=2|2-4|=4$

$\therefore \lim_{x\to 2^-} f(x)=f(2)=\lim_{x\to 2^+} f(x)$

$\Rightarrow$ f is continuous at $x=2$ ✱

(b)



11. $y=e^{-2x}\sin 3x$ — $(\frac{d}{dx})$

$\therefore \frac{dy}{dx}=-2(y)+e^{-2x}(3\cos 3x)$

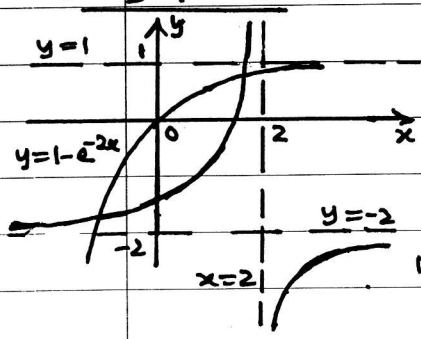
$=3e^{-2x}\cos 3x-2y$ ✱

$\frac{d^2y}{dx^2}=-2\left[\frac{dy}{dx}+2y\right]+3e^{-2x}(-3\sin 3x)-2\frac{dy}{dx}$

$=-13y-4\frac{dy}{dx}$ ✱

12. $y=\frac{3-2x}{x-2}=-2-\frac{1}{x-2}$

$\therefore$ asymptotes : $y=-2, x=2$ ✱



For intersection pt:

$\frac{3-2x}{x-2}=1-e^{-2x}$

$e^{-2x}=\frac{3x-5}{x-2}$

$\therefore x-2=(3x-5)e^{2x}$ ✱

No. of real roots $=2$ ✱

Let $f(x)=(3x-5)e^{2x}-x+2$

$f'(x)=3e^{2x}+2(3x-5)e^{2x}-1$

$=(6x-7)e^{2x}-1$

By NRM: $x_0=1.7$

$x_1=1.7-\frac{f(1.7)}{f'(1.7)}=1.7-\frac{3.2964}{94.885}$

$=1.67$ (3s.f) ✱

1. $3\cos 2x-\sin x-2=0 \Rightarrow 3(1-2s^2)-s-2=0$

$\therefore 6s^2+s-1=0 \Rightarrow (3s-1)(2s+1)=0$

$\therefore \sin x=\frac{1}{3}$ or $-\frac{1}{2}$

$\therefore x=0.3, 2.8, 3.7$ or 5.8 ✱

2. $\tan\theta+\cot\theta=\frac{5}{2}$: $\tan^2\theta+2+\cot^2\theta=\left(\frac{5}{2}\right)^2$

$\therefore \tan^2\theta+\cot^2\theta=\frac{17}{4}$ — ①

$\therefore \frac{s}{c}+\frac{c}{s}=\frac{5}{2}$: $\frac{s^2+c^2}{cs}=\frac{5}{2}=\sec\theta.\operatorname{cosec}\theta$ — ②

①+②: $\tan^2\theta+\sec\theta.\operatorname{cosec}\theta+\cot^2\theta=\frac{27}{4}$ ✱

3. $\frac{dy}{dx}=y^2e^x \Rightarrow \int_1^y \frac{1}{y^2}dy=\int_0^x e^x dx$

$\therefore \left[-\frac{1}{y}\right]_1^y=\left[e^x\right]_0^x \Rightarrow -\frac{1}{y}+1=e^x-1$

$\therefore y=\frac{1}{2-e^x}$ ✱

4. (a) $\underline{x} \perp \underline{y}$: $\begin{pmatrix}2\\3\end{pmatrix}\cdot\begin{pmatrix}k\\-4\end{pmatrix}=0$

$\therefore 2k-12=0 \Rightarrow k=6$ ✱

(b) $\theta=60°$: $\underline{x}\cdot\underline{y}=|\underline{x}||\underline{y}|\cos 60°$

$\therefore 2k-12=\sqrt{13}\cdot\sqrt{k^2+16}\left(\frac{1}{2}\right)$

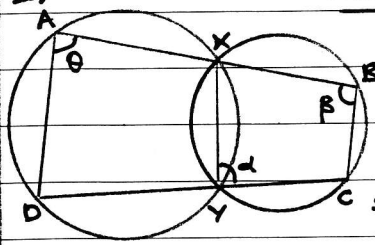
$\therefore 4(4k^2-48k+144)=13(k^2+16)$

$\therefore 3k^2-192k+368=0$

$\therefore k=\frac{192\pm\sqrt{(-192)^2-4(3)(368)}}{2(3)}$

$=\frac{192\pm 104\sqrt{3}}{6}=\frac{1}{3}[96\pm 52\sqrt{3}]$ ✱

5.



Statements	Reasons
$\theta=\alpha$	Exterior angle of cyclic quadrilateral ADYX.
$\alpha+\beta=\pi$	Sum of opposite interior angles of cyclic quad BCYX.
$\therefore \theta+\beta=\pi$ — ①	①: A pair of supplementary interior angles.
$\therefore AD // BC$ ✱	

6. $\underline{V}_R=\underline{V}_C+\underline{V}_S$

V_S: $5ms^{-1}$ to NW

$|\underline{V}_C|=25ms^{-1}$

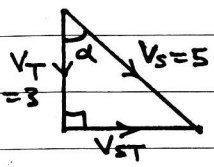
$\underline{V}_R$: to east.

$\frac{\sin\theta}{5}=\frac{\sin 125°}{25} \Rightarrow \theta=9.8°$

$\therefore$ Direction of cannon = to S 80.2° E ✱

$\underline{V}_T=3ms^{-1}$ to S , $|\underline{V}_S|=5ms^{-1}$

to intercept: $\underline{V}_{ST}$: to E , $\underline{V}_{ST}=\underline{V}_S-\underline{V}_T$



$\cos\alpha=\frac{3}{5} \Rightarrow \alpha=53.1°$

$\therefore$ course of warship

= to S 53.1° E ✱

LEE HOCK LEONG
B. Sc (Ed.) - 1st. Class Hons.
M. Sc (Pure Maths.) - CGPA4

7. (i) $P(4Y) = \frac{{}^4C_4}{{}^9C_4} = \frac{1}{126}$ ✱

(ii) Prob. $= \frac{1}{{}^9C_4}\left({}^3C_1 \cdot {}^4C_1 \cdot {}^2C_2 + {}^3C_1 \cdot {}^4C_2 \cdot {}^2C_1 + {}^3C_2 \cdot {}^4C_1 \cdot {}^2C_1\right) = \frac{72}{126} = \frac{4}{7}$ ✱

8. (i) $F(1) = 1$: $3k = 1 \Rightarrow k = \frac{1}{3}$ ✱

(ii) $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} \frac{1}{3} & -1 \le x < 0 \\ \frac{2}{3} & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$ ✱

(iii) $\mu = E(X) = \int_{-1}^{0} \frac{1}{3}x\,dx + \int_0^1 \frac{2}{3}x\,dx$

$= \frac{1}{6}[x^2]_{-1}^{0} + \frac{1}{3}[x^2]_0^1 = \frac{1}{6}(0-1) + \frac{1}{3}(1-0) = \frac{1}{6}$ ✱

(iv) $E(X^2) = \int_{-1}^{0} \frac{1}{3}x^2\,dx + \int_0^1 \frac{2}{3}x^2\,dx$

$= \frac{1}{9}[x^3]_{-1}^{0} + \frac{2}{9}[x^3]_0^1 = \frac{1}{9}(0+1) + \frac{2}{9}(1-0) = \frac{1}{3}$

$\therefore \sigma = \left[\frac{1}{3} - \left(\frac{1}{6}\right)^2\right]^{1/2} = \frac{\sqrt{11}}{6}$ ✱

(v) $P(|X - \frac{1}{6}| > \frac{1}{3})$

$= P(X - \frac{1}{6} < -\frac{1}{3}) + P(X - \frac{1}{6} > \frac{1}{3})$

$= F(-\frac{1}{6}) + [1 - F(\frac{1}{2})]$

$= \frac{1}{3}[-\frac{1}{6} + 1] + 1 - \frac{1}{3}[2(\frac{1}{2}) + 1] = \frac{11}{18}$ ✱

9. X = no. of person infected.

For $X \sim B(5, 0.1)$: $P(X=0) = 0.9^5 = 0.5905$ ✱

For $X \sim B(50, 0.1)$: $X \sim P_o(5)$

$P(X<4) = e^{-5}\left(1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!}\right) = 0.2650$ ✱

For $X \sim B(n, 0.1)$: $P(X \le n-1) > 0.99985$

$\therefore 1 - P(X=n) > 0.99985 \Rightarrow 0.00015 > 0.1^n$

$\therefore n(-1) < \lg 0.00015 \Rightarrow n > 3.82$

$\therefore n_{min} = 4$ ✱

10. $S \sim N(20, 1.2^2)$, $L \sim N(k, 1.8^2)$

(a) $X = \sum_1^8 S_i$: $X \sim N(8(20), 8(1.2)^2)$

$P(X > 170) = P\left(Z > \frac{170-160}{1.2\sqrt{8}}\right)$

$= P(Z > 2.946) = 0.0016$ ✱

(b) $Y = \sum_1^4 L_i$: $Y \sim N(4k, 4(1.8)^2)$

$E(X-Y) = 160 - 4k$ ✱

$Var(X-Y) = 8(1.2)^2 + (-1)^2 4(1.8)^2 = 24.48$ ✱

(c) $P(X > Y) = 0.9$: $P(X - Y > 0) = 0.9$

$\therefore P\left(Z > \frac{0-(160-4k)}{\sqrt{24.48}}\right) = 0.9$

$\therefore 4k - 160 = (-1.282)\sqrt{24.48}$

$\therefore k = 38.41 \approx 38$ ✱

(d) $2X - Y \sim N\left(2(160) - 152, 2^2(8)(1.2)^2 + 4(1.8)^2\right)$

$P(2X > Y + 150) = P(2X - Y > 150)$

$= P\left(Z > \frac{150-168}{\sqrt{59.04}}\right) = P(Z > -2.3426)$

$= 0.9904$ ✱

11. (a) $\sum P(X=x) = 1$: $2k + k + 3k + 4k = 1$

$\therefore k = 0.1$ ✱

(b) $E(X) = 0 + 1(0.1) + 2(0.3) + 3(0.4) = 1.9$ ✱

$E(X^2) = 0 + 1^2(0.1) + 2^2(0.3) + 3^2(0.4) = 4.9$

$\therefore Var(X) = 4.9 - 1.9^2 = 1.29$ ✱

(c) $E(2X-1) = 2(1.9) - 1 = 2.8$ ✱

$Var(2X-1) = 2^2(1.29) = 5.16$ ✱

$F(x) = P(X \le x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \le x < 1 \\ 0.3 & 1 \le x < 2 \\ 0.6 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$ ✱

12. (a) $n = 50$, $\sum fx = 880$, $\sum fx^2 = 17230$

$\bar{x} = \frac{880}{50} = 17.6$ ✱

$s^2 = \frac{17230}{50} - 17.6^2 = 34.84$ ✱

(b) $m = x_{25} = 15.5 + \left(\frac{25-16}{18}\right)(5) = 18$

$SIQR = \frac{1}{2}(Q_3 - Q_1) = \frac{1}{2}(x_{37.5} - x_{12.5})$

$= \frac{1}{2}\left[20.5 + \left(\frac{37.5-34}{12}\right)(5) - 10.5 - \left(\frac{12.5-7}{9}\right)(5)\right]$

$= 4.20$ ✱

(c) Percentage $= \left(\frac{12+4}{50}\right) \times 100\%$

$= 32\%$ ✱

LEE HOCK LEONG
B. Sc (Ed.) - 1st. Class Hons.
M. Sc (Pure Maths.) - CGPA4