SMJK PEREMPUAN CINA (2005)
MATHEMATICS T / S (PAPER 1)

1. Express $\sqrt{98-40\sqrt{6}}$ in the form $a\sqrt{2}+b\sqrt{3}$ where a and b are integers. [4]

2. Find the modulus and argument of the complex number $(1-\sqrt{2}\,i)^3$. [6]

3. Find the set of values of x such that $\dfrac{|x|+1}{|x|-3}<4$. [8]

4. Use the trapezium rule to obtain the approximate value of $\displaystyle\int_0^1 \frac{1}{1+\sqrt{2x}}\,dx$ by using five ordinates. [8]

5. Sketch the graph of f in the domain $-3 \le x \le 5$ where

   $f(x) = 2 - x^2$, $-3 \le x < 0$ and $f(x) = f(x-2)$, $0 < x \le 5$.

   Show that f is not continuous at $x = 0$. [8]

6. Given that $(x+2)$ is a factor of $f(x) = x^3 + (a+2b)x^2 + (a-3b)x + 8$.
   Find a in terms of b.

   Find $q(x)$ such that $f(x) = (x+2).q(x)$ holds for all values of b.

   Determine the set of values of b such that $f(x) = 0$ has at least two distinct real roots. [8]

7. Show that $x^4 - x = 10$ has only one root between 1 and 2.

   Hence, find this root correct to three significant figures. [8]

8. By expressing $f(x)$ where $f(x) = \dfrac{22-3x}{6-x-2x^2}$ in partial fractions, expand $f(x)$ as a series in ascending powers of x up to and include the term in x^2. [6]

9. (a) By using the substitution $u = \sqrt{2+x}$, find the value of $\displaystyle\int_2^7 \frac{1}{x\sqrt{2+x}}\,dx$. [6]

   (b) Solve $\displaystyle\int e^{2x}\sin x\,dx$ using integration by parts. [6]

10. Matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 2 & 4 \\ 0 & 4 & 4 \\ -2 & 2 & 6 \end{pmatrix}$.

    Find $\mathbf{A}^2 - 12\mathbf{A} + 44\mathbf{I}$ where $\mathbf{I}$ is the 3 x 3 identity matrix.

    Show that $\mathbf{A}(\mathbf{A}^2 - 12\mathbf{A} + 44\mathbf{I}) = 48\mathbf{I}$. Hence, deduce the value of $\mathbf{A}^{-1}$. [8]

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11. (a) State the equation of the asymptotes of the curve $y = \dfrac{x-3}{(x-2)(x+1)}$. [2]

    (b) Find the points where the curve intersect the axes, and find the stationary points of the curve. [4]

    (c) Sketch the curve. [4]

    (d) Determine the set of values of k for which the equation $x - 3 = k(x-2)(x+1)$ does not have any real roots. [2]

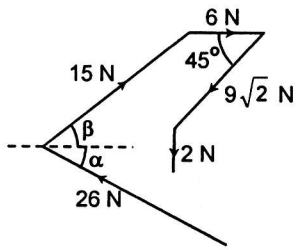
12. Two parallel sides of a rectangle are each increasing in length at the rate of 2 cms^{-1}, while the other two parallel sides shorten such that the rectangle has a constant area of 50 cm^3. Given that, at the time t, the length of each lengthening side is x, and the length of each shortening side is y, and the perimeter of the rectangle is p,

    (a) show that $\dfrac{dp}{dt} = 4\left(1 - \dfrac{y}{x}\right)$. [4]

    (b) find the rate of change in the perimeter when
    (i) $x = 5$ cm [2]
    (ii) $y = 5$ cm [2]

    (c) Find the value of x and y when the perimeter of the rectangle is minimum. [4]

MATHEMATICS T (PAPER 2)

1. 

   Five forces of magnitudes 15 N, 26 N, $9\sqrt{2}$ N, 2 N and 6 N act on an object as shown in the figure.
   If $\tan\alpha = \dfrac{5}{12}$ and $\tan\beta = \dfrac{3}{4}$, find the magnitude and direction of the resultant force on the object. [5]

2. In a triangle PQR, PQ = a, QR = $a + d$, RP = $a + 2d$ and $\angle PQR > 120°$, with a and d as positive constants. Show that $\dfrac{2}{3}a < d < a$. [5]

3. By taking the cube of the function $\cos^2\theta + \sin^2\theta$, show that
   $\cos^6\theta + \sin^6\theta = \dfrac{1}{8}(5 + 3\cos 4\theta)$. [3]

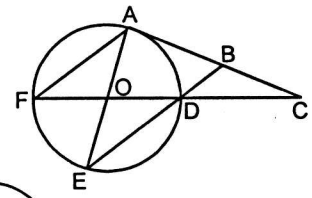
   Hence,
   (a) determine the maximum and minimum values of $\cos^6\theta + \sin^6\theta$ for all values of θ. [2]

   (b) solve the equation $2(\cos^6\theta + \sin^6\theta) = \sin 4\theta$ for $0° \le \theta \le 90°$. [5]

4. (a) In the figure, FODC is a straight line and O is the centre of the circle ADEF.
If FD = DC and $\angle BDC = \theta$,

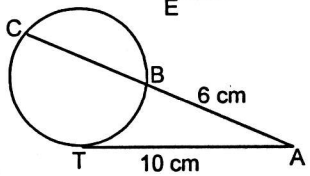
(i) find $\angle AFO$ in terms of θ. [3]

(ii) prove that AB = BC. [2]



(b) In the figure, TA is tangent to the circle at T.

Given that TA = 10 cm, AB = 6 cm, calculate length of the chord BC. [4]



5. In a chemical process to produced substance P from the substance Q, each molecule of Q is changed into one molecule of P. The rate of increase in the number of molecules of P at any instant is described by the differential equation $\frac{dm}{dt} = a(N - m)$, where m is the number of molecules of P at the time t minutes after the start of the process, a is a positive constant, and N is the molecules of Q at the start of the process.

(i) If $m = 0$ when $t = 0$, solve the differential equation and sketch its graph. [7]

(ii) If the number of molecules of P and Q are equal when $t = 50$, determine the length of time elapses before the number of molecules of P exceeds 0.99 N. [6]

6. At 1400 hours, two cars are at distances 300 m and 400 m from the point of intersection O of two straight roads crossing at a right angle and are approaching O at uniform speeds 20 ms^{-1} and 40 ms^{-1} respectively. Find the shortest distance between the cars and the time taken to reach this point. [8]

7. The following data show the number of students enter the public library A and public library B each hour from 10.00 am to 9 pm, on a certain day.

Public library A : 14, 31, 2, 25, 9, 21, 19, 28, 16, 17, 10
Public library B : 3, 5, 6, 7, 8, 10, 14, 21, 19, 12, 22

Draw two box plots to compare the number of students that enter the library A and library B hourly. Comments on your results. [6]

8. Ten people go to the cinema and sit in a group of ten reserved seats in the front row. If three of the ten people are sisters and must be seated together,

(a) find the number of possible seating arrangements for all ten people, [2]

(b) find the number of possible seating arrangements for all ten people, if two of the remaining seven refuse to sit together. [3]

9. A survey shows that 60% of the housewives have seen the advertisement of a new product in television. The probability that the housewife who has seen the advertisement buys the new product is 0.9, while the probability that the housewife who has not seen the advertisement buys the new product is 0.3.

(i) Find the probability that a housewife buys the new product. [3]

(ii) Find the probability that a housewife who buys the new product has seen the advertisement. [3]

(iii) Using an appropriate approximation, find the probability that less than 10 out of 50 housewives who buys the new product, have not seen the advertisement. [4]

10. A continuous random variable T has probability density function

$$f(t) = \begin{cases} \frac{t}{a} + 1 & , -a \le t < 0 \\ e^{-2t} & , t \ge 0 \\ 0 & , \text{otherwise} \end{cases}$$, where a is a positive constant.

(i) Find the value of a, and sketch the graph of f. [4]

(ii) Find the expected value of T. [3]

(iii) Find the cumulative distribution function, and the median of T. [5]

(iv) Find the value of b such that $P(T \ge b) = 2\,P(T < b)$. [3]

11. In an examination, the average score was 70 and the standard deviation was 8, and the scores follows a normal distribution.

(a) If 12% are given grade A's, what is the lowest possible score for grade A and the highest possible for grade B, to the nearest integer. [3]

(b) If all the students who scored 72 marks up to 77 marks are given a B grade, and there are 10 students obtained B grade, determine the total number of students who sat for the examination. [3]

12. The ages of 120 college students and teachers are grouped in the following table,

Age (years)	18 – 22	23 – 27	28 – 32	33 – 37	38 – 42	43 – 47	48 – 52
No. Of students & teacher	20	25	30	21	11	8	5

(a) Estimate the median of the above frequency distribution by using

(i) an ogive [4]

(ii) a non-graphical method. [3]

(b) Find the mean age. [1]

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1. Let $\sqrt{98-40\sqrt{6}} = a\sqrt{2}+b\sqrt{3}$:

$\therefore\ 98-40\sqrt{6} = 2a^2+3b^2+2ab\sqrt{6}$

$\therefore\ 2a^2+3b^2=98$ and $ab=-20$

$\therefore\ 2a^2+3\left(\frac{400}{a^2}\right)=98 \Rightarrow a^4-49a^2+600=0$

$(a^2-24)(a^2-25)=0 \Rightarrow a=\pm 5\ (\because a\in\mathbb{Z})$

$a=5, b=-4$ or $a=-5, b=4$

$\therefore\ \sqrt{98-40\sqrt{6}} = 5\sqrt{2}-4\sqrt{3}$ ✱ $[\because \sqrt{A}\geq 0]$

2. $z=(1-\sqrt{2}i)^3 = 1+3(-\sqrt{2}i)+3(-\sqrt{2}i)^2+(-\sqrt{2}i)^3$

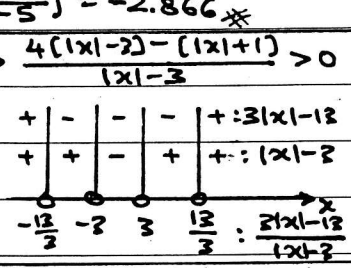
$= 1-3\sqrt{2}i-6+2\sqrt{2}i = -5-\sqrt{2}i$

$|z| = \sqrt{25+2} = 3\sqrt{3}$ ✱

$\arg(z) = \tan^{-1}\left[\frac{-\sqrt{2}}{-5}\right] = -2.866$ ✱

3. $\frac{|x|+1}{|x|-3} < 4 \Rightarrow \frac{4(|x|-3)-(|x|+1)}{|x|-3} > 0$

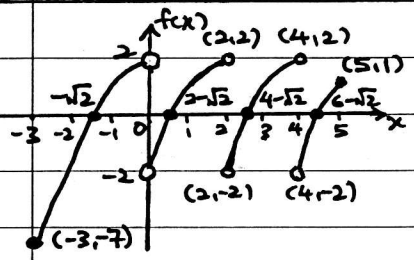
$\therefore\ \frac{3|x|-13}{|x|-3} > 0$



$x\in\left(-\infty,-\frac{13}{3}\right)\cup(-3,3)\cup\left(\frac{13}{3},\infty\right)$ ✱

4. $\int_0^1 \frac{1}{1+\sqrt{5x}}\,dx = \frac{1}{2}\left(\frac{1}{4}\right)\left[1+\frac{1}{1+\sqrt{2}}+2\left(\frac{1}{1+\sqrt{0.5}}+\frac{1}{1+1}+\frac{1}{1+\sqrt{1.5}}\right)\right] = 0.5606$ (4 d.p) ✱

5.



since $f(0)$ is undefined $\therefore f$ is not continuous at $x=0$ ✱

6. $(x+2)$ factor : $f(-2)=0$

$-8+4(a+2b)-2(a-3b)+8=0 \Rightarrow a=-7b$ ✱

$f(x) = x^3-5bx^2-10bx+8 = (x+2)[x^2-(5b+2)x+4]$

$\therefore\ q(x) = x^2-(5b+2)x+4$ ✱

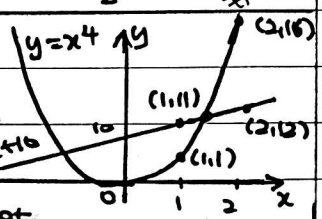
For $f(x)=0$ to have at least 2 distinct real roots $\Rightarrow B^2-4AC\geq 0$ and $q(-2)\neq 0$

$\therefore\ (5b+2)^2-16\geq 0$ and $4+2(5b+2)+4\neq 0$

$\therefore\ (5b+6)(5b-2)\geq 0$ and $b\neq -\frac{6}{5}$

$\therefore\ \left\{b : b<-\frac{6}{5} \text{ or } \frac{2}{5}\leq b\right\}$ ✱

7. $x^4-x=10 \Rightarrow x^4=x+10$



exists only 1 intersection point between $x=1$ and $x=2$

$\therefore\ x^4-x=10$ has only 1 root between $x=1$ and $x=2$. ✱

Let $f(x)=x^4-x-10$: $f'(x)=4x^3-1$

By NRM : $x_{n+1} = x_n - f(x_n)/f'(x_n)$

Let $x_0=1.85$: $x_1 = 1.85 - \frac{(-0.1365)}{24.327}$

$= 1.856 \approx 1.86$

$x_2 = 1.856 - \frac{0.01021}{24.574} = 1.856 \approx 1.86$

$\therefore$ root $= 1.86$ (2 s.f) ✱

8. $f(x) = \frac{22-3x}{6-x-2x^2} \equiv \frac{A}{3-2x}+\frac{B}{2+x} = \frac{A(2+x)+B(3-2x)}{6-x-2x^2}$

$\therefore\ 22-3x = A(2+x)+B(3-2x)$

$x=-2$: $28=B(7) \Rightarrow B=4$, $[x]$: $-3=A-2B \Rightarrow A=5$

$\therefore\ f(x) = \frac{4}{2+x}+\frac{5}{3-2x} = 2\left(1+\frac{x}{2}\right)^{-1}+\frac{5}{3}\left(1-\frac{2}{3}x\right)^{-1}$

$= 2\left[1+\left(-\frac{x}{2}\right)+\left(-\frac{x}{2}\right)^2+\dots\right]+\frac{5}{3}\left[1+\frac{2}{3}x+\left(\frac{2}{3}x\right)^2+\dots\right]$

$= \frac{11}{3}+\frac{1}{9}x+\frac{67}{54}x^2+\dots$ ✱

9.(a) $u^2=2+x \Rightarrow 2u\frac{du}{dx}=1$

$\int_2^7 \frac{dx}{x\sqrt{2+x}} = \int_2^3 \frac{2u\,du}{(u^2-2)u} = \int_2^3 \frac{2\,du}{u^2-2}$

$= \frac{1}{\sqrt{2}}\int_2^3 \left[\frac{1}{u-\sqrt{2}}-\frac{1}{u+\sqrt{2}}\right]du$

$= \frac{1}{\sqrt{2}}\left[\ln\left(\frac{u-\sqrt{2}}{u+\sqrt{2}}\right)\right]_2^3$

$= \frac{1}{\sqrt{2}}\left[\ln\left(\frac{3-\sqrt{2}}{3+\sqrt{2}}\right)-\ln\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)\right]$

$= \frac{1}{\sqrt{2}}\ln\left[\frac{(3-\sqrt{2})(2+\sqrt{2})}{(3+\sqrt{2})(2-\sqrt{2})}\right]$

$= \frac{1}{\sqrt{2}}\ln\left[\frac{4+\sqrt{2}}{4-\sqrt{2}}\cdot\frac{4+\sqrt{2}}{4+\sqrt{2}}\right] = \frac{1}{\sqrt{2}}\ln\left(\frac{9+4\sqrt{2}}{7}\right)$ ✱

(b) Let $I=\int e^{2x}\sin x\,dx$:

$\therefore\ I = e^{2x}(-\cos x)-\int 2e^{2x}(-\cos x)\,dx$

$\therefore\ I = -e^{2x}\cos x+\left[2e^{2x}(\sin x)-\int 4e^{2x}\sin x\,dx\right]$

$\therefore\ 5I = e^{2x}(2\sin x-\cos x)$

$\therefore\ I = \frac{1}{5}e^{2x}[2\sin x-\cos x]+c$ ✱

10. $A^2 = \begin{pmatrix}2&2&4\\0&4&4\\-2&2&6\end{pmatrix}\begin{pmatrix}2&2&4\\0&4&4\\-2&2&6\end{pmatrix} = \begin{pmatrix}-4&20&40\\-8&24&40\\-16&16&36\end{pmatrix}$

$\therefore\ A^2-12A+44I = \begin{pmatrix}-4&20&40\\-8&24&40\\-16&16&36\end{pmatrix}-\begin{pmatrix}24&24&48\\0&48&48\\-24&24&72\end{pmatrix}+\begin{pmatrix}44&0&0\\0&44&0\\0&0&44\end{pmatrix}$

$= \begin{pmatrix}16&-4&-8\\-8&20&-8\\8&-8&8\end{pmatrix}$ ✱

$\therefore\ A(A^2-12A+44I) = \begin{pmatrix}2&2&4\\0&4&4\\-2&2&6\end{pmatrix}\begin{pmatrix}16&-4&-8\\-8&20&-8\\8&-8&8\end{pmatrix}$

$= \begin{pmatrix}48&0&0\\0&48&0\\0&0&48\end{pmatrix} = 48I$ ✱

$\therefore\ A^{-1} = \frac{1}{48}(A^2-12A+44I) = \frac{1}{12}\begin{pmatrix}4&-1&-2\\-2&5&-2\\2&-2&2\end{pmatrix}$ ✱

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11. (a) asymptotes: $y=0,\ x=2,\ x=-1$ ✱

(b) x-axis: $(y=0) \Rightarrow x-3=0$

$\therefore x=3 \Rightarrow (3,0)$ ✱

y-axis: $(x=0)$ $y=\frac{-3}{(-2)(1)}=\frac{3}{2}$ $\therefore (0,\frac{3}{2})$ ✱

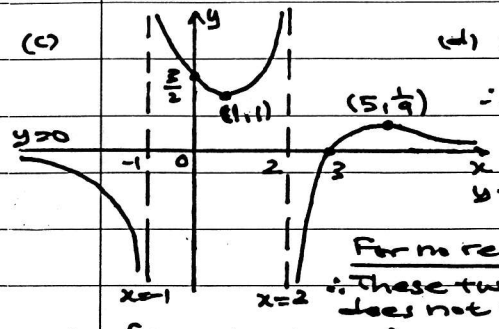
$y=\frac{x-3}{(x-2)(x+1)} \Rightarrow y'=\frac{(x-3)(2x-1)-(x^2-x-2)}{(x-2)^2(x+1)^2}$

For s.points: $y'=0 \Rightarrow x^2-6x+5=0$

$\therefore (x-1)(x-5)=0 \Rightarrow x=1$ or 5

$\therefore$ s.points $=(1,1)$ and $(5,\frac{1}{9})$ ✱

(c)



(d) $x-3=k(x-2)(x+1)$

$\therefore \frac{x-3}{(x-2)(x+1)}=k$

$\therefore y=k$ and $y=\frac{x-3}{(x-2)(x+1)}$

For no real roots:

$\therefore$ These two graphs does not intersects.

$\therefore \{k : \frac{1}{9}<k<1\}$ ✱

12. $\frac{dx}{dt}=2$, $xy=50 \Rightarrow y=\frac{50}{x}$

(a) $P=2x+2y$: $\frac{dP}{dt}=2\frac{dx}{dt}+2\frac{dy}{dt}$, $\frac{dy}{dt}=-\frac{50}{x^2}\cdot\frac{dx}{dt}$

$\therefore \frac{dP}{dt}=4+2\left[-\frac{1}{x}\left(\frac{50}{x}\right)(2)\right]=4\left[1-\frac{y}{x}\right]$ ✱

(b) $x=5$: $y=10 \Rightarrow \frac{dP}{dt}=4(1-\frac{10}{5})=-4\,cms^{-1}$ ✱

$y=5$: $x=10 \Rightarrow \frac{dP}{dt}=4(1-\frac{5}{10})=2\,cms^{-1}$ ✱

(c) For P_{min}: $\frac{dP}{dt}=0 \Rightarrow y=x$

$\therefore x^2=50 \Rightarrow x=5\sqrt{2}$ $(\because x>0)$

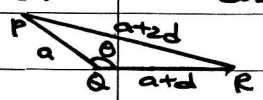
$\therefore x=5\sqrt{2}\,cm=y$ ✱

1. $F_R=\begin{pmatrix}15\cos\beta\\15\sin\beta\end{pmatrix}+6\begin{pmatrix}1\\0\end{pmatrix}+9\sqrt{2}\begin{pmatrix}-\cos45^\circ\\-\sin45^\circ\end{pmatrix}$

$+2\begin{pmatrix}0\\-1\end{pmatrix}+26\begin{pmatrix}-\cos\alpha\\\sin\alpha\end{pmatrix}=\begin{pmatrix}-15\\-2\end{pmatrix}$

$=\sqrt{229}$ N, parallel to $\begin{pmatrix}-15\\-2\end{pmatrix}$ ✱

2.



$\cos\theta=\frac{a^2+(a+d)^2-(a+2d)^2}{2a(a+d)}$

$=\frac{a^2-2ad-3d^2}{2a(a+d)}$

$=\frac{(a+d)(a-3d)}{2a(a+d)}=\frac{a-3d}{2a}$

For $\theta>120^\circ$: $-1<\cos\theta<-\frac{1}{2}$

$\therefore -1<\frac{a-3d}{2a}<-\frac{1}{2} \Rightarrow -2a<a-3d<-a$

$\therefore -3a<-3d<-2a \Rightarrow \frac{2}{3}a<d<a$ ✱

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3. $(c^2+s^2)^3=c^6+3c^4s^2+3c^2s^4+s^6=1$

$\therefore c^6+s^6=1-3c^2s^2(c^2+s^2)$

$=1-3\left[\frac{\sin2\theta}{2}\right]^2(1)=1-\frac{3}{4}\left[\frac{1-\cos4\theta}{2}\right]$

$\therefore \cos^6\theta+\sin^6\theta=\frac{1}{8}[5+3\cos4\theta]$ ✱

(a) $-3\le3\cos4\theta\le3$, $\forall\theta\in\mathbb{R}$

$\therefore \frac{1}{8}(5-3)\le\frac{1}{8}(5+3\cos4\theta)\le\frac{1}{8}(5+3)$

$\therefore (\cos^6\theta+\sin^6\theta)_{max}=1$ ✱

$(\cos^6\theta+\sin^6\theta)_{min}=\frac{1}{4}$ ✱

(b) $2[\cos^6\theta+\sin^6\theta]=\sin4\theta=\frac{2}{8}[5+3\cos4\theta]$

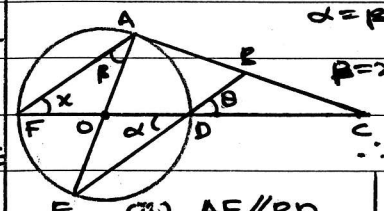
$\therefore 4\sin4\theta-3\cos4\theta=5$ [let $\tan2\theta=t$]

$\therefore 4\left(\frac{2t}{1+t^2}\right)-3\left(\frac{1-t^2}{1+t^2}\right)=5 \Rightarrow t^2-4t+4=0$

$\therefore (t-2)^2=0 \Rightarrow \tan2\theta=2$

$\therefore 2\theta=63.43^\circ \Rightarrow \theta=31.7^\circ$ ✱

4(a) (i)



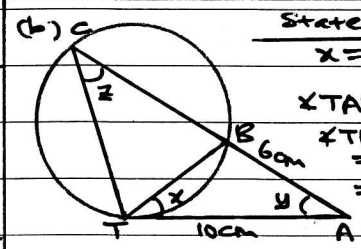
Statements	Reason
$\theta=\alpha$	Opposite angles
$\alpha=\beta$	Angles in the same segment.
$\beta=x$	$OA=OF=r$ $\Rightarrow \Delta OAF$ is isoscales

$\therefore \angle AFO=x=\beta=\alpha=\theta$ ✱

(ii)

Statements	Reason
$AF \parallel BD$	$x=\theta$ (corresponding angles)
$\therefore \angle FAC=\angle DBC$, $\angle FCA=\angle DCB$	Corresponding angles common angles.
$\therefore \Delta FAC \sim \Delta DBC$	
$\Rightarrow \frac{AC}{BC}=\frac{FC}{DC}=2$	Given $FD=DC$
$\therefore AB+BC=2BC$	
$\therefore AB=BC$ ✱	

(b)



Statements	Reasons
$x=z$	Angles of alternate segment.
$\angle TAB=\angle CAT$	common angle
$\angle TBA=\pi-x-y$	sum of angles in Δ
$=\pi-z-y$	
$=\angle CTA$	sum of angles in Δ

$\therefore \Delta TBA \sim \Delta CTA$

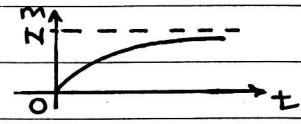
$\therefore \frac{TA}{BA}=\frac{CA}{TA} \Rightarrow (6+BC)=\frac{10^2}{6} \Rightarrow BC=\frac{32}{3}$ cm ✱

5. $\frac{dm}{dt}=a(N-m) \Rightarrow \int_0^m\frac{dm}{N-m}=\int_0^t a\,dt$

$\therefore [\ln(N-m)]_0^m=-a[t]_0^t \Rightarrow \ln\left(\frac{N-m}{N}\right)=-at$

$\therefore 1-\frac{m}{N}=e^{-at}$

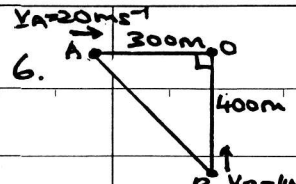
$\therefore m=N[1-e^{-at}]$ ✱



(ii) $t=50$, $m=\frac{N}{2}$: $-a(50)=\ln(\frac{1}{2}) \Rightarrow a=\frac{1}{50}\ln2$

For $m>0.99N$: $t>-\frac{50}{\ln2}\ln(0.01)=332.19$

$\therefore$ time elapse ≈ 322 minutes ✱

6. 

$\underline{V}_{BA} = \begin{pmatrix}0\\40\end{pmatrix} - \begin{pmatrix}20\\0\end{pmatrix} = \begin{pmatrix}-20\\40\end{pmatrix}$

$\underline{S}_{BA} = \begin{pmatrix}300\\-400\end{pmatrix} + t\begin{pmatrix}-20\\40\end{pmatrix}$

For S.Distance: $\underline{V}_{BA} \cdot \underline{S}_{BA} = 0$

$\begin{pmatrix}-20\\40\end{pmatrix} \cdot \begin{pmatrix}300-20t\\-400+40t\end{pmatrix} = 0$

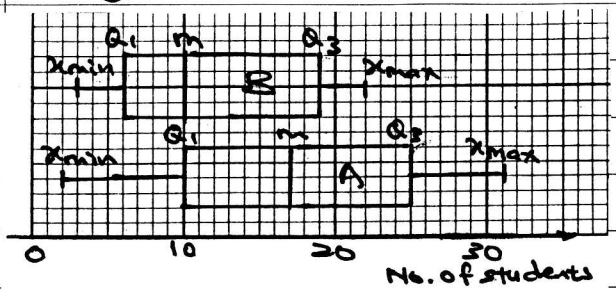
$\therefore -6000 + 400t - 16000 + 1600t = 0 \Rightarrow t = 11s$ ✱

$\therefore$ S.Distance $= \left|\begin{pmatrix}300-20(11)\\-400+40(11)\end{pmatrix}\right| = \left|40\begin{pmatrix}2\\1\end{pmatrix}\right|$

$= 40\sqrt{5}\,m$ ✱

7. A: 2, 9, 10, 14, 16, 17, 19, 21, 25, 28, 31

B: 3, 5, 6, 7, 8, 10, 12, 14, 19, 21, 22



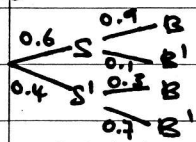
There are more students going to Library A than Library B. ✱
A: distributed almost symmetric
B: skewed right (positive) ✱

8.(a) No. of ways $= 8!(3!) = 241920$ ✱

(b) No. of ways $= 8!(3!) - 7!(3!)(2!)$

$= 181440$ ✱

9. Let S = seen the ad. B = buy the product



(i) $P(B) = 0.6(0.9) + 0.4(0.8)$

$= 0.66$ ✱

(ii) $P(S|B) = \dfrac{P(S \cap B)}{P(B)} = \dfrac{0.6(0.9)}{0.66} = \dfrac{9}{11}$ ✱

(iii) Let X = no. of housewives who buys, have not seen the ad.

$\therefore X \sim B(50, \frac{2}{11}) \rightarrow X \sim N(\frac{100}{11}, \frac{900}{121})$

$P(X<10) \rightarrow P(X<9.5)$

$= P\left(Z < \dfrac{9.5 - \frac{100}{11}}{\sqrt{\frac{900}{121}}}\right) = P(Z<0.15)$

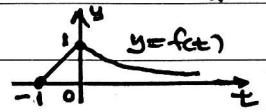
$= 0.5596$

10.(i) $\int_{-\infty}^{\infty} f(t)\,dt = 1$: $\int_{-a}^{0}\left(\frac{t}{a}+1\right)dt + \int_0^{\infty} e^{-2t}\,dt = 1$

$\therefore \left[\frac{t^2}{2a} + t\right]_{-a}^{0} - \frac{1}{2}\left[e^{-2t}\right]_0^{\infty} = 1$

$\therefore 0 - \left(\frac{a}{2} - a\right) - \frac{1}{2}[0-1] = 1 \Rightarrow a = 1$ ✱

(ii) $E(T) = \int_{-\infty}^{\infty} t \cdot f(t)\,dt$



$= \int_{-1}^{0}(t^2+t)\,dt + \int_0^{\infty} t \cdot e^{-2t}\,dt$

$= \left[\frac{t^3}{3} + \frac{t^2}{2}\right]_{-1}^{0} + \left[t \cdot \frac{e^{-2t}}{-2}\right]_0^{\infty} + \int_0^{\infty}\frac{1}{2}e^{-2t}\,dt$

$= 0 - \left[-\frac{1}{3} + \frac{1}{2}\right] + [0-0] - \frac{1}{4}\left[e^{-2t}\right]_0^{\infty} = \frac{1}{12}$ ✱

(iii) $\underline{-1 \le t < 0}$: $P(T \le t) = \int_{-1}^{t}(t+1)\,dt$

$= \left[\frac{(t+1)^2}{2}\right]_{-1}^{t} = \frac{1}{2}(t+1)^2$

$\underline{0 \le t}$: $P(T \le t) = \frac{1}{2}(1)^2 + \int_0^t e^{-2t}\,dt$

$= \frac{1}{2} - \frac{1}{2}\left[e^{-2t}\right]_0^t = \frac{1}{2} - \frac{1}{2}\left[e^{-2t} - 1\right] = \frac{1}{2}\left[2 - e^{-2t}\right]$

$\therefore F(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{2}(t+1)^2 & -1 \le t < 0 \\ \frac{1}{2}(2 - e^{-2t} & 0 \le t \end{cases}$

since $F(0) = \frac{1}{2} \Rightarrow m = 0$ ✱

(iv) $P(T \ge b) = 2P(T<b) = 1 - P(T<b)$

$\therefore P(T<b) = \frac{1}{3} = \frac{1}{2}(b+1)^2$

$\therefore b+1 = \sqrt{\frac{2}{3}} \Rightarrow b = \sqrt{\frac{2}{3}} - 1$ ✱ $[\because b+1>0]$

11. Let X = marks obtained $X \sim N(70, 8^2)$

(a) $P(X > A_{min}) = 0.12 = P\left(Z > \frac{A_{min} - 70}{8}\right)$

$\therefore \frac{A_{min} - 70}{8} = z_{0.12} = 1.175$

$\therefore A_{min} = 79.4 \approx 79$ ✱ $\therefore B_{max} = 78$ ✱

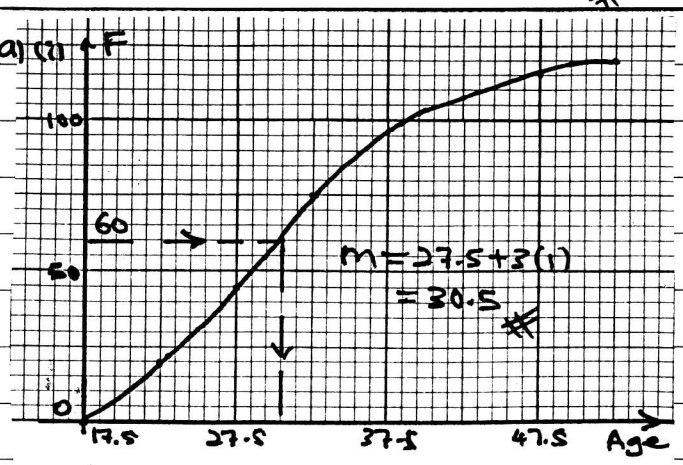
(b) $P(72 < X < 77) = P\left(\frac{72-70}{8} < Z < \frac{77-70}{8}\right)$

$= P(Z > 0.25) - P(Z > 0.875)$

$= 0.4013 - 0.1908 = 0.2105$

$\therefore 0.2105 = \frac{10}{N} \Rightarrow N = 47.51 \approx 48$ ✱

12.(a)(i)



(ii) $m = x_{60} = 27.5 + \left(\frac{60-45}{30}\right)(5) = 30$ ✱

(b) $\bar{x} = \dfrac{\sum fx}{n}$ (x = midpoints)

$= \dfrac{3710}{120} = 30.92$ years. ✱

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