SMJK HUA LIAN, TAIPING (2005)
MATHEMATICS T (PAPER 1)

1. Given that $5x + 4yi = (4y - 5xi)^2$. Find the values of x and y. [5]

2. A curve has parametric equations $x = 2t - \ln 2t$, $y = t^2 - \ln t^2$, where $t > 0$.
Find the value of t at the point on the curve at which the gradient is 2. [5]

3. Sketch the curve $xy = 6$ and the line $y = 9 - 3x$ in the first quadrant.
Find the area bounded by the two curves. [6]

4. Prove that $2\left(1+\frac{1}{15}\right)^{-\frac{1}{4}} = 15^{\frac{1}{4}}$.

Find the expansion of $(1 + x)^{-\frac{1}{4}}$ in ascending powers of x until the term in x^3.

By substituting $x = \frac{1}{15}$, find the value of $15^{\frac{1}{4}}$, giving your answer correct to five decimal places. [7]

5. The polynomial $p(x) = x^4 + ax^3 + bx^2 - 2x - 4$ has factors $(x - 1)$ and $(x + 2)$.
   (i) Find the values of a and b.
   (ii) Find all the roots for which $p(x) = 0$.
   (iii) Find the set of values of x for which $p(x)$ is positive. [8]

6. Find the following integrals.
   (i) $\int \sin^2 3x \, dx$ (ii) $\int \ln(x + 2) \, dx$ (iii) $\int \frac{1}{e^x - 4e^{-x}} dx$ [8]

7. The line $y = mx + c$ intersects the parabola $y^2 = 4ax$ at the point P and Q.
Show that the coordinates of the mid-point of PQ is $\left(\frac{2a - mc}{m^2}, \frac{2a}{m}\right)$.
If the mid-point is M, find the locus of M as m varies with $c = 1$. [10]

8. Find the coordinates of the turning points on the curve $y = x^3 - 12x - 12$ and sketch the graph.
Find the set of values of k for which the equation $x^3 - 12x - 12 = k$ has more than one real roots.
By taking 4 as the first approximation, use the Newton-Raphson method to obtain a sequence of approximations for the positive root of the equation $x^3 - 12x - 12 = 0$ correct to three decimal places. [12]

9. Express $\frac{1}{r(r+1)(r+3)}$ in partial fractions.
Find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+3)}$ and deduce the sum to infinity. [8]

10. Matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 1 & 1 \\ 0 & 1 & -2 \end{pmatrix}$.
   (i) Find the matrix **B** such that $\mathbf{B} = \mathbf{A}^2 - 10\mathbf{I}$, where $\mathbf{I}$ is the 3 x 3 identity matrix.
   (ii) Find $(\mathbf{A} + \mathbf{I})\mathbf{B}$ and hence find $(\mathbf{A} + \mathbf{I})^{25}\mathbf{B}$. [9]

11. The function f is defined by $f(x) = \begin{cases} \ln|x+1| - 1 & , x \le -2 \\ \frac{|x|}{x} & , x > -2 \text{ and } x \ne 0. \\ 1 & , x = 0 \end{cases}$
   (i) Sketch the graph of f.
   (ii) Determine whether f is continuous at $x = -2$ and $x = 0$. Justify your answers. [10]

12. Given that $y = \frac{3x-5}{x^2-1}$, prove that if x is real then y cannot takes values between $\frac{1}{2}$ and $\frac{9}{2}$.
Write the equations of the asymptotes for $y = \frac{3x-5}{x^2-1}$.
Find the turning points and sketch the graph. [12]

MATHEMATICS T (PAPER 2)

1. Solve the differential equation $\frac{dy}{dx} = \frac{y(2x+y)}{x(y-x)}$, given that $y = 2$ when $x = 1$. [5]

2. Prove the cosine using the vector method.
Hence, with the usual notation in a triangle XYZ, $x = 7$, $y = 4$, $z = 5$, find $\cos X$ as a fraction in its lowest term and prove that $\sin Y = \frac{8\sqrt{6}}{35}$.
Find also the shortest altitude of the triangle. [7]

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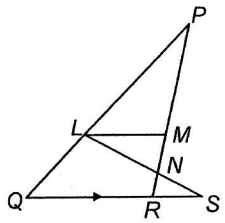
3. Given that $PL = QS = 18$ cm, $LM = LQ$, $RS = 10$ cm, $LN = 6$cm and LM is parallel to QS.



(a) Name a triangle that is congruent to triangle PLM and give your reasons.

(b) Show that triangle PLN is similar to triangle SRN.

(c) Calculate the length of RN.

(d) Let LM be x cm and forms an equation for the similar triangles PLM and PQR. Show that it can reduces to $x^2 + 18x - 144 = 0$.

(e) Find the length of LM. [11]

4. At a particular instant, the position vector of particles A and B are $2\underline{i}$ m and $3\underline{i}$ m respectively. Particle A moves with velocity $(8\underline{i} + 6\underline{j})$ ms^{-1} and particle B moves with velocity $(15\underline{i} + 20\underline{j})$ ms^{-1}. Find

(a) the magnitude and direction of the velocity of particle B relative to particle B.

(b) the time when B is due east of A.

(c) the shortest distance between the two particles and the time that elapses before the two particles are at the shortest distance apart from each other. [12]

5. Prove that $\cos\theta - \cos 3\theta = 4\sin^2\theta\cos\theta$.
Hence, find the solution of the equation $\cos\theta - \cos 3\theta = \tan^2\theta$, given that $0 \le \theta \le 2\pi$. [6]

6. In a model of mortgage repayment, the sum of money owed to the bank is x at time t. The sum of money owed to the bank increases, due to interest, at a rate proportional to the sum of the money owed. Money is also repaid at a constant rate r.

When $x = a$, interest and repayment balance. Show that $\frac{dx}{dt} = \frac{r}{a}(x - a)$.

If $x = A$ when $t = 0$, find x in terms of t, r, a and A.
Sketch the graphs of x against t in two cases (i) $A > a$, (ii) $A < a$.
State the circumferences under which the loan is repaid in a finite time T, and show that in this case, $T = \frac{a}{r}\ln\left(\frac{a}{a - A}\right)$. [10]

7. (a) In the letters of the word 'UNIVERSITY' are arranged in a line at random, what is the probability that arrangement with two 'I's are separated.

(b) The probabilities of the events A and B are given as $P(A) = \frac{5}{12}$, $P(A \cap B) = \frac{1}{6}$, $P(A' \cap B') = 1 - q$. Find, in terms of q,

(i) $P(B)$ (ii) $P(A|B)$

(iii) the value of q, given that A and B are independent events. [7]

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8. Given that both the values of the mean and standard deviation of the random variable $X + 2$ are μ. Find the value of μ such that the value of expectation of X^2 is minimum. [4]

9. A piece of machinery breaks down at an average of once a fortnight and these breakdowns occur throughout a week.

(a) Find the probability that in a week there are no breakdowns.

(b) Find the probability that one of the two consecutive weeks with no breakdowns and on with one breakdown.

(c) A week with no breakdowns is called a star week. Every 10 weeks the number of star weeks is recorded and a report is sent to the machine manufacturer. Find the probability that there are more than 8 star weeks in the next report. [8]

10. In the production of compact discs at a certain factory, the proportion of faulty discs is known to be $\frac{1}{5}$. Each week the factory produces 1000 discs.

Estimate, to 2 significant figures, the probability that there will be at most 220 faulty discs produced in one week.

It costs 50 cents to produced a disc. A faulty disc has to be discarded, while a non-faulty disc is sold at RM 1.20. Find the expected profit made by the factory per week. [8]

11. The cumulative distribution function of the continuous random variable X is given by

$$F(x) = \begin{cases} 0 & , x \le 1 \\ k(x-1)^3 & , 1 < x \le 3 \\ 1 & , 3 < x \end{cases}$$. Calculate the value of k.

(i) Find median of X, giving your answer correct to 3 significant figures.

(ii) Sketch the graph of the probability density function of X.

(iii) Find the probability density function of Y, if $Y = X^2$. [12]

12. The scores of a statistics test of 120 students are summarized in the table.

Scores	No. of students
10 – 19	2
20 – 29	6
30 – 39	12
40 – 49	20
50 – 59	36
60 – 69	28
70 – 79	11
80 – 89	4
90 – 99	1

(i) Plot a cumulative relative frequency curve for the data.

(ii) From the curve drawn, estimate the median and semi-interquartile range test marks.

(iii) The passing mark is 45 and the minimum mark for grade A is 78.
What is the probability that a randomly selected student who passed the test, obtained grade A. [10]

1. $5x+4yi = (4y-5xi)^2$

$\therefore (4y-5xi)^2 - i(4y-5xi) = 0$

$\therefore (4y-5xi)(4y-(5x+1)i) = 0$

$\therefore 4y-5xi = 0$ or $4y-(5x+1)i = 0$

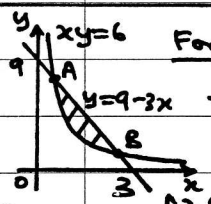
$\therefore x=0, y=0$ or $y=0, x=-\frac{1}{5}$ ✱

2. $x = 2t - \ln 2t \Rightarrow \frac{dx}{dt} = 2 - \frac{1}{t} = \frac{2t-1}{t}$

$y = t^2 - \ln t^2 \Rightarrow \frac{dy}{dt} = 2t - \frac{2}{t} = \frac{2(t^2-1)}{t}$

$m=2$: $\frac{2(t^2-1)}{2t-1} = 2 \Rightarrow t^2-2t=0$

$(\because t \neq 0)$: $t=2$ ✱

3. 

For A, B: $x(9-3x)=6$

$\therefore x^2-3x+2=0 = (x-1)(x-2)$

$\therefore x=1$ or 2

$\therefore$ Area $= \int_1^2 [9-3x-\frac{6}{x}]dx$

$= [9x-\frac{3}{2}x^2-6\ln x]_1^2 = 9(2-1)-\frac{3}{2}(4-1)-6\ln 2$

$= \frac{9}{2} - 6\ln 2$ ✱

4. $2(1+\frac{1}{15})^{-\frac{1}{4}} = 2(\frac{15}{16})^{1/4} = 15^{1/4}$ ✱

$f(x) = (1+x)^{-\frac{1}{4}} = 1 + (-\frac{1}{4})x + \frac{(-\frac{1}{4})(-\frac{5}{4})}{2!}x^2 + \frac{(-\frac{1}{4})(-\frac{5}{4})(-\frac{9}{4})}{3!}x^3 + \ldots = 1 - \frac{1}{4}x + \frac{5}{32}x^2 - \frac{15}{128}x^3 + \ldots$ ✱

$\therefore (1+\frac{1}{15})^{-\frac{1}{4}} = f(\frac{1}{15}) \Rightarrow (15)^{1/4} = 2.f(\frac{1}{15})$

$\therefore 15^{\frac{1}{4}} = 2[1-\frac{1}{4}(\frac{1}{15}) + \frac{5}{32}(\frac{1}{15})^2 - \frac{15}{128}(\frac{1}{15})^3 + \ldots]$

$= 1.96799$ (5dp) ✱

5. (i) $P(1)=0$: $1+a+b-2-4=0$

$\therefore a+b=5$ — ①

$P(-2)=0$: $16-8a+4b+4-4=0 \Rightarrow 2a-b=4$ — ②

① + ②: $3a=9 \Rightarrow a=3$ $\therefore b=2$ ✱

(ii) $P(x) = x^4+3x^3+2x^2-2x-4$

$= (x-1)(x+2)(x^2+2x+2)$

$\therefore$ For $P(x)=0$: roots $= 1, -2, \frac{-2\pm\sqrt{4-8}}{2(1)}$

$\therefore$ roots $= 1, -2, -1\pm i$ ✱

(iii) For $P(x)>0$: $(x-1)(x+2)>0$ $[\because x^2+2x+2>0 \ \forall x \in \mathbb{R}]$

$\therefore \{x: x<-2 \text{ or } 1<x\}$ ✱

6. (i) $\int \sin^2 3x\,dx = \frac{1}{2}\int[1-\cos 6x]dx$

$= \frac{1}{2}[x - \frac{1}{6}\sin 6x] + c$ ✱

(ii) $\int \ln(x+2)\,dx = \int \ln u\,du$ $(u=x+2)$

$= u\ln u - \int u(\frac{1}{u})du = u\ln u - u + c$

$= (x+2)[\ln(x+2)-1] + c$ ✱

(iii) $\int \frac{1}{e^x-4e^{-x}}dx = \int \frac{1}{y^2-4}dy$ $[y=e^x]$

$= \frac{1}{4}\int[\frac{1}{y-2} - \frac{1}{y+2}]dy$

$= \frac{1}{4}\ln(\frac{y-2}{y+2}) + c = \frac{1}{4}\ln[\frac{e^x-2}{e^x+2}] + c$ ✱

7. For P, Q: $y^2 = 4a[\frac{y-c}{m}]$

$\therefore y^2 - \frac{4a}{m}y + \frac{4ac}{m} = 0$

$(\because y_P$ and y_Q are roots$]$: $y_P + y_Q = \frac{4a}{m}$

Let $M(\alpha,\beta) = M_{PQ}$: $\beta = \frac{y_P+y_Q}{2} = \frac{2a}{m}$

M lies on $y=mx+c$: $\alpha = \frac{1}{m}[\beta - c]$

$\therefore \alpha = \frac{1}{m}[\frac{2a}{m} - c] = \frac{2a-mc}{m^2}$

$\therefore M = (\frac{2a-mc}{m^2}, \frac{2a}{m})$ ✱

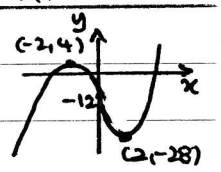
For locus M $(c=1)$: $x=\alpha$, $y=\beta$

$\therefore y = \frac{2a}{m}$, $x = \frac{2a-m}{m^2}$

$\therefore x(\frac{2a}{y})^2 = 2a - (\frac{2a}{y})$

$\therefore 2ax = y^2 - y$ [locus of M] ✱

8. $y = x^3-12x-12 \Rightarrow y' = 3x^2-12$



$y'=0$: $3(x^2-4)=0 \Rightarrow x=\pm 2$

$\therefore$ turning points $= (2,-28), (-2,4)$ ✱

For $x^3-12x-12=k$ has more than one real roots $\Rightarrow$ exists intersection between graphs of $y=x^3-12x-12$ and $y=k$.

$\therefore \{k: -28 \le k \le 4\}$ ✱

$f(x)=y$: $f'(x)=y'$, By NRM:

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_0=4$: $x_1 = 4 - \frac{4}{36} = 3.8889$

$x_2 = 3.8889 - \frac{0.14715}{33.3706} = 3.88449 \approx 3.884$

$x_3 = 3.88449 - \frac{0.00021}{33.2678} = 3.88448 \approx 3.884$

$\therefore$ root $= 3.884$ (3dp) ✱

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9. $f(r) = \dfrac{1}{r(r+1)(r+3)} \equiv \dfrac{A}{r} + \dfrac{B}{r+1} + \dfrac{C}{r+3}$

$\equiv \dfrac{A(r+1)(r+3) + Br(r+3) + Cr(r+1)}{r(r+1)(r+3)}$

$\therefore 1 = A(r+1)(r+3) + Br(r+3) + Cr(r+1)$

$r=0$: $1 = A(3) \Rightarrow A = \frac{1}{3}$

$r=-1$: $1 = B(-2) \Rightarrow B = -\frac{1}{2}$

$r=-3$: $1 = C(6) \Rightarrow C = \frac{1}{6}$

$\therefore f(r) = \dfrac{1}{3r} - \dfrac{1}{2(r+1)} + \dfrac{1}{6(r+3)}$ ✱

$\sum_{r=1}^{n} f(r) = \sum_{r=1}^{n}\left[\frac{1}{3}\left(\frac{1}{r} - \frac{1}{r+1}\right) + \frac{1}{6}\left(\frac{1}{r+3} - \frac{1}{r+1}\right)\right]$

$= \frac{1}{3}\left[\left(1+\frac{1}{2}+\dots+\frac{1}{n}\right) - \left(\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{n}+\frac{1}{n+1}\right)\right]$

$+ \frac{1}{6}\left[\left(\frac{1}{4}+\frac{1}{5}+\dots+\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}\right) - \left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots\right.\right.$

$\left.\left.+\frac{1}{n+1}\right)\right] = \frac{1}{3}\left(1 - \frac{1}{n+1}\right) + \frac{1}{6}\left(\frac{1}{n+2} + \frac{1}{n+3} - \frac{1}{2} - \frac{1}{3}\right)$

$= \frac{7}{36} - \frac{1}{3(n+1)} + \frac{1}{6}\left(\frac{1}{n+2} + \frac{1}{n+3}\right)$ ✱

$\sum_{r=1}^{\infty} f(r) = \lim_{n\to\infty}\left[\sum_{r=1}^{n} f(r)\right]$

$= \frac{7}{36} - 0 + \frac{1}{6}(0+0) = \frac{7}{36}$ ✱

10. (i) $B = A^2 - 10I = \begin{bmatrix}1&2&-3\\3&1&1\\0&1&-2\end{bmatrix}\begin{bmatrix}1&2&-3\\3&1&1\\0&1&-2\end{bmatrix} - 10I$

$= \begin{bmatrix}7&1&5\\6&8&-10\\3&-1&5\end{bmatrix} - \begin{bmatrix}10&0&0\\0&10&0\\0&0&10\end{bmatrix} = \begin{bmatrix}-3&1&5\\6&-2&-10\\3&-1&-5\end{bmatrix}$ ✱

(ii) $(A+I)B = \begin{bmatrix}2&2&-3\\3&2&1\\0&1&-1\end{bmatrix}\begin{bmatrix}-3&1&5\\6&-2&-10\\3&-1&-5\end{bmatrix} = \begin{bmatrix}-3&1&5\\6&-2&-10\\3&-1&-5\end{bmatrix}$ ✱

Let $A+I=P$: $PB = B$

$\therefore P^2B = P\,PB = PB = B$

$\therefore P^4B = P^2P^2B = P^2B = B$

$\therefore P^8B = P^4.P^4B = P^4B = B$

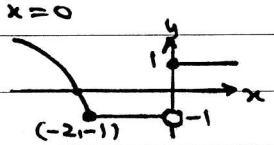
$\therefore P^{25}B = P.P^8.P^8.P^8B = P.P^8.P^8B$

$= P.P^8B = PB = B$

$\therefore (A+I)^{25}B = B = \begin{bmatrix}-3&1&5\\6&-2&-10\\3&-1&-5\end{bmatrix}$ ✱

11. (i) $f(x) = \begin{cases} \ln(x+1) - 1, & x \le -2 \\ \frac{|x|}{x}, & x > -2, x \ne 0 \\ 1, & x = 0 \end{cases}$

$= \begin{cases} \ln(-x-1) - 1, & x \le -2 \\ -1, & -2 < x < 0 \\ 1, & 0 \le x \end{cases}$



(ii) Graph of f is joint at $x=-2$

$\Rightarrow f$ is continuous at $x=-2$ ✱

Graph of f is disjoint at $x=0$

$\Rightarrow f$ is not continuous at $x=0$ ✱

12. $y = \dfrac{3x-5}{x^2-1} \Rightarrow yx^2 - y = 3x - 5 \quad (x \ne \pm 1)$

$\therefore yx^2 - 3x + 5 - y = 0$

For real x: $B^2 - 4AC \ge 0 \Rightarrow 9 - 4y(5-y) \ge 0$

$\therefore 4y^2 - 20y + 9 \ge 0 \Rightarrow (2y-1)(2y-9) \ge 0$

$\therefore y \le \frac{1}{2}$ or $\frac{9}{2} \le y$

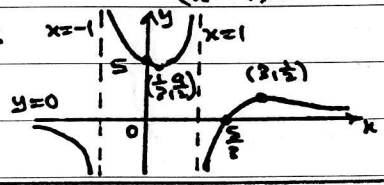
$\therefore y \notin \left(\frac{1}{2}, \frac{9}{2}\right)$ ✱ asymptotes: $x = \pm 1, y = 0$ ✱

$\dfrac{dy}{dx} = \dfrac{(x^2-1)(3) - (3x-5)(2x)}{(x^2-1)^2} = \dfrac{-(3x-1)(x-3)}{(x^2-1)^2}$

$\dfrac{dy}{dx} = 0$: $x = \frac{1}{3}$ or 3

$\therefore$ turning points

$= \left(\frac{1}{3}, \frac{9}{2}\right), \left(3, \frac{1}{2}\right)$



1. Let $y = ux$: $\dfrac{dy}{dx} = \dfrac{y(2x+y)}{x(y-x)}$

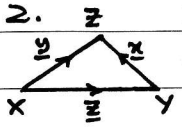
$\therefore u + x\dfrac{du}{dx} = \dfrac{u(2+u)}{u-1} \Rightarrow x\dfrac{du}{dx} = \dfrac{3u}{u-1}$

$\therefore \int_2^u \left(1 - \frac{1}{u}\right)du = \int_1^x \frac{3}{x}\,dx \Rightarrow [u - \ln u]_2^u = [3\ln x]_1^x$

$\therefore u - 2 - \ln\left(\frac{u}{2}\right) = 3\ln x \Rightarrow \frac{y}{x} - 2 = \ln\left[x^3\left(\frac{y}{2x}\right)\right]$

$\therefore x^2y = 2e^{\left(\frac{y-2x}{x}\right)}$ ✱

2.



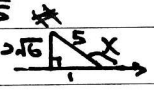
$\underline{x}.\underline{x} = (\underline{y} - \underline{z}).(\underline{y} - \underline{z})$

$|\underline{x}|^2 = |\underline{y}|^2 + |\underline{z}|^2 - \underline{z}.\underline{y} - \underline{y}.\underline{z} \quad (\underline{z}.\underline{y} = \underline{y}.\underline{z})$

$= |\underline{y}|^2 + |\underline{z}|^2 - 2|\underline{y}||\underline{z}|\cos X$ ✱

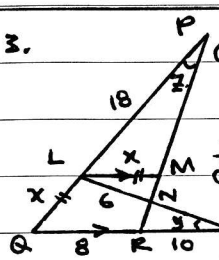
$x=7, y=4, z=5$: $\cos X = \dfrac{4^2+5^2-7^2}{2(4)(5)} = -\frac{1}{5}$ ✱

$\therefore \sin Y = \frac{y}{x}\sin X = \frac{4}{7}\left(\frac{2\sqrt{6}}{5}\right) = \frac{8\sqrt{6}}{35}$ ✱



Shortest altitude $= z.\sin Y = \dfrac{8\sqrt{6}}{7}$ ✱

3.



Statements	Reasons
(a) $PL = SQ$, $LM = QL$	Given
$\angle PLM = \angle SQR$	Corresponding Angles
$\therefore \Delta PLM \equiv \Delta SQR$	By SAS
(b) $z = y$	$\Delta PLM \equiv \Delta SQR$
$\angle PNL = \angle SNR$	opposite angles
$\angle PLN = \pi - z - \angle PNL$ $= \pi - y - \angle SNR = \angle SRN$	Sum of angles in Δ.
$\therefore \Delta PLN \sim \Delta SRN$ ✱	

(c) $\dfrac{RN}{LN} = \dfrac{SR}{PL} \Rightarrow RN = 6\left(\frac{10}{18}\right) = \frac{10}{3}$ cm

(d) $\angle PLM = \angle PQR$, $\angle PML = \angle PRQ$ } Corresponding angles; $\angle LPM = \angle QPR$ (common)

$\therefore \Delta PLM \sim \Delta PQR$: $\dfrac{LM}{QR} = \dfrac{PL}{PQ} \Rightarrow x(x+18) = 8(18)$

$\therefore x^2 + 18x - 144 = 0$ ✱

(e) $(x-6)(x+24) = 0 \Rightarrow x = 6$ ✱ $(\because x > 0)$

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4. (a) $\underline{v}_{BA} = \begin{pmatrix}15\\20\end{pmatrix} - \begin{pmatrix}8\\6\end{pmatrix} = \begin{pmatrix}7\\14\end{pmatrix} = 7\begin{pmatrix}1\\2\end{pmatrix}$

$= 7\sqrt{5}\,ms^{-1}$, to N 26.6° E

(b) $\underline{s}_{BA} = \left[\begin{pmatrix}3\\0\end{pmatrix} - \begin{pmatrix}0\\2\end{pmatrix}\right] + t\begin{pmatrix}7\\14\end{pmatrix} = \begin{pmatrix}3\\-2\end{pmatrix} + t\begin{pmatrix}7\\14\end{pmatrix}$

B due east of A: $\underline{s}_{BA} = k\begin{pmatrix}1\\0\end{pmatrix}$

$\therefore -2 + 14t = 0 \Rightarrow t = \frac{1}{7}$ s

(c) For shortest distance: $\underline{s}_{BA} \cdot \underline{v}_{BA} = 0$

$\therefore 7(3-4) + 49t(1+4) = 0 \Rightarrow t = \frac{1}{35}$ s

$\therefore$ Shortest Distance $= \left|\begin{pmatrix}3\\-2\end{pmatrix} + \frac{1}{35}\begin{pmatrix}7\\14\end{pmatrix}\right| = \left|\begin{matrix}3.2\\-1.6\end{matrix}\right|$

$= \frac{8}{5}\sqrt{5}$ m

5. $\cos\theta - \cos 3\theta = c - [4c^3 - 3c] = 4c(1-c^2)$

$= 4\sin^2\theta\cos\theta$

$\cos\theta - \cos 3\theta = \tan^2\theta \Rightarrow 4s^2c = \frac{s^2}{c^2}$

$\therefore s^2[4c^3 - 1] = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{1}{\sqrt[3]{4}}$

$\therefore \theta = 0, \pi, 2\pi, 0.8893, 5.3939$

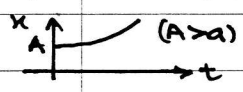
6. $\frac{dx}{dt} = kx - r$: $(x = a)$ $\frac{dx}{dt} = 0 \Rightarrow k = \frac{r}{a}$

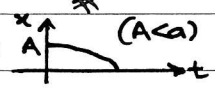
$\therefore \frac{dx}{dt} = \frac{rx}{a} - r = \frac{r}{a}(x-a)$

$\therefore \int_A^x \frac{dx}{x-a} = \int_0^t \frac{r}{a}\,dt \Rightarrow [\ln(x-a)]_A^x = \frac{r}{a}[t]_0^t$

$\therefore \ln\left(\frac{x-a}{A-a}\right) = \frac{r}{a}[t] \Rightarrow \frac{x-a}{A-a} = e^{rt/a}$

$\therefore x = a + (A-a)e^{rt/a}$

(i) 

(ii) 

For loan repaid: $A < a$

$x = 0$: $\frac{r}{a}T = \ln\left(\frac{a}{a-A}\right) \Rightarrow T = \frac{a}{r}\ln\left(\frac{a}{a-A}\right)$

7. (a) Prob. $= \frac{8!\,({}^9C_3)}{\left(\frac{10!}{2!}\right)} = 0.8$

(b) (i) $P(A' \cap B') = 1 - q = P(A \cup B)'$

$\therefore P(A \cup B) = q = \frac{5}{12} + P(B) - \frac{1}{6}$

$\therefore P(B) = q - \frac{1}{4}$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{6} \Big/ \frac{1}{4}(4q-1) = \frac{2}{3(4q-1)}$

(iii) A, B independent: $\frac{5}{12} = \frac{2}{3(4q-1)} \Rightarrow q = \frac{13}{20}$

8. $Var(X) = E(X^2) - [E(X)]^2$: $E(X^2) = \mu^2 + (\mu - 2)^2$

$\therefore E(X^2) = 2\mu^2 - 4\mu + 4 = 2(\mu-1)^2 + 2$

$\therefore E(X^2)_{min.} = 2$ when $\mu = 1$

9. X = no. of breakdowns in a week, $X \sim Po(\frac{1}{2})$

(a) $P(X=0) = e^{-0.5} = 0.6065$

(b) Prob. $= P(X=0) \cdot P(X=1) \times 2$

$= e^{-0.5} \cdot e^{-0.5}(0.5) \times 2 = 0.3679$

(c) Prob. $= {}^{10}C_9(e^{-0.5})^9(1-e^{-0.5}) + (e^{-0.5})^{10} = 0.0504$

10. X = no. of faulty discs. $X \sim B(1000, 0.2)$

$X \sim N(200, 160)$, $np = 200\ (>5)$, $n = 1000\ (>50)$

$P(X \le 220) \to P(X < 220.5) = P\left(Z < \frac{220.5 - 200}{\sqrt{160}}\right)$

$= P(Z < 1.6207) = 0.9475$

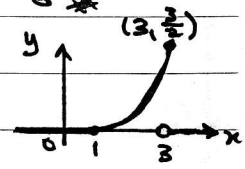
$E(\text{profit}) = 1.2(1000 - 200) - 1000(0.5)$

$= \text{RM}460$

11. $F(3) = 1$: $k(2)^3 = 1 \Rightarrow k = \frac{1}{8}$

(i) $F(m) = \frac{1}{2}$: $\frac{1}{8}(m-1)^3 = \frac{1}{2}$

$\therefore m = 1 + \sqrt[3]{4}$



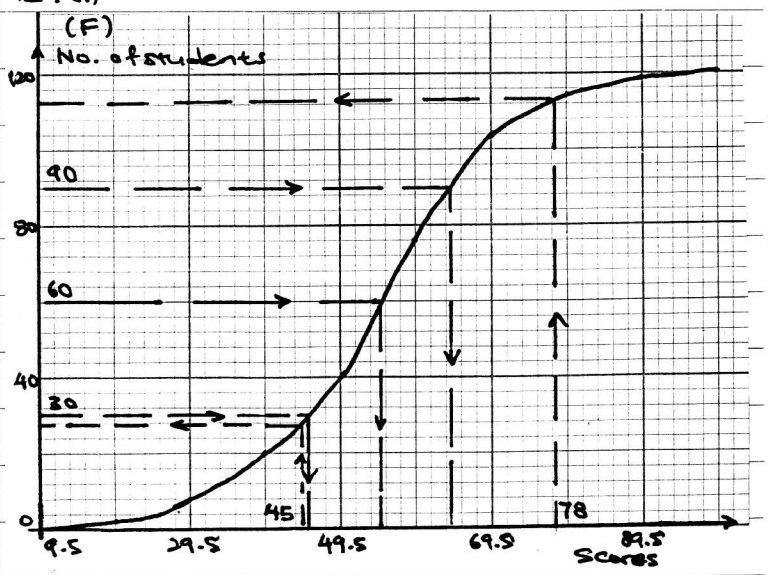
(ii) $f(x) = \frac{d}{dx}(F(x)) = \begin{cases} \frac{3}{8}(x-1)^2, & 1 < x \le 3 \\ 0, & \text{otherwise} \end{cases}$

(iii) $Y = X^2$: $P(Y \le y) = P(X^2 \le y) = P(X \le \sqrt{y})$

$(\because x > 0)$ $= \frac{1}{8}(\sqrt{y} - 1)^3$, $1 < y \le 9$

$\therefore g(y) = \frac{d}{dy}P(Y \le y) = \begin{cases} \frac{3}{16\sqrt{y}}(\sqrt{y}-1)^2, & 1 < y \le 9 \\ 0, & \text{otherwise} \end{cases}$

12. (i)



(ii) $m = 54.9$

$SIQR = \frac{1}{2}[64.1 - 45.5] = 9.3$

(iii) $P(X \ge 78 | X \ge 45) = \frac{8}{92} = 0.087$

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