SM DATO ONN (2005)
MATHEMATICS T / S (PAPER 1)

1. Solve the equation $2\log_x 3 - \log_3\sqrt{x} = \frac{3}{2}$. [6]

2. Determine the value of a if $\frac{\sqrt{2}+ai}{1+\sqrt{2}i}$ is a real number, and find this real number. [4]

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3. Given that $\mathbf{P} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$.

   (i) Find **R** such that $\mathbf{R} = \mathbf{P}^2 - 4\mathbf{P} - \mathbf{I}$. [3]

   (ii) Show that $\mathbf{PR} + 4\mathbf{I} = \mathbf{0}$. [2]

4. Evaluate the definite integral $\int_0^1 x(x^2+1)^4\,dx$. [6]

5. A curve has parametric equations $x = e^{2t} - 2t$ and $y = e^t + t$.
   Find the gradient of the curve at the point with $t = \ln 2$. [5]

6. The function f is defined as $f: x \to 4 + (x-1)^2, x \in \mathcal{R}$.

   (i) Sketch the graph of f. [3]

   (ii) State the range of f. [2]

   (iii) Determine whether f^{-1} exists. Give your reasons. [2]

7. The sum of the first n terms of a series is $1 - \left(\frac{3}{4}\right)^n$.

   Show that the terms in this series are in geometric progression, and find the first term and the common ratio of this series. [7]

8. (a) Find $\frac{dy}{dx}$ where $y = \sqrt{\frac{x+1}{x-1}}$. [5]

   (b) Find the gradient of the curve $y^3 - 2xy + 3x^2 - 3 = 0$ at the point (2, 3). [5]

9. If $\frac{y}{x}$ is small enough for powers of $\frac{y}{x}$ higher than the third to be neglected,

   show that $\sqrt{x+y} - \sqrt{x-y} = \sqrt{x}\left(\frac{y}{x} + \frac{y^3}{8x^3}\right)$.

   Use this result to deduce a rational approximation to $\sqrt{6} - \sqrt{2}$.

   Deduce a rational approximation to $\sqrt{6} + \sqrt{2}$. [10]

10. Show that $x - 1$ is a factor of the polynomial $4x^3 - 19x^2 + 11x + 4$. [2]

    Hence, find al the real roots of the equation $4x^6 - 19x^4 + 11x^2 + 4 = 0$. [4]

    Determine the set of values of x such that $4x^3 - 19x^2 + 11x + 4 < 20 - 20x$. [4]

11. Show that the curve $y = \frac{\ln x}{x}$ has a stationary point at $\left(e, \frac{1}{e}\right)$, and determine whether this point is a local minimum or a local maximum point. [6]

    Sketch the curve. [4]

    Show that the area of the region bounded by the curve $y = \frac{\ln x}{x}$, the x–axis, and the line $x = \frac{1}{e}$ is equal to the area of the region bounded by the curve $y = \frac{\ln x}{x}$, x–axis, and the line $x = e$. [5]

12. Show that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

    $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

    Deduce that both roots are real if $b^2 - 4ac \geq 0$, and are complex if $b^2 - 4ac < 0$. [4]

    Determine all real values of k for which the quadratic equation
    $x^2 - (k-3)x + k^2 + 2k + 5 = 0$ has real roots. [5]

    If α and β are the roots of this equation, show that $\alpha^2 + \beta^2 = -(k+5)^2 + 24$.
    Hence, find the maximum value for $\alpha^2 + \beta^2$. [6]

MATHEMATICS T (PAPER 2)

1. It is given that $\tan A = 2$. Find, without using trigonometrical tables or a calculator

   (i) the exact value of $\tan\theta$, given that $\tan(\theta + A) = 4$. [2]

   (ii) the exact value of $\tan\phi$, given that $\sin(\phi + A) = 3\cos(\phi - A)$. [3]

2. Find all the values of θ, where $-\pi \leq \theta \leq \pi$, which satisfy the equation

   $\sin 4\theta - \sin 2\theta = \cos 3\theta$. [4]

3. The points A, B and C are three points on the horizontal ground with B due north of A, and the bearing of C from B being 060°. The angles of elevation of the top of a vertical tower situated at B from A and C are both β. The point P lies on AC such that AP : PC = 1 : 2. Show that the angle of elevation of the top of the tower from P is $\tan^{-1}(\sqrt{3}\tan\beta)$. [8]

4. Eleven cards each bear a single letter, and together they can be made to spell the word "EXAMINATION". Three cards are selected from the eleven cards, and the order or selection is not relevant. Find how many possible selections can be made

(i) if the three cards all bear different letters,

(ii) if two of the three cards bear the same letter. [4]

5. Solve the differential equation $x^2y^2\frac{dy}{dx} = y^2 - 4$, given that $x = 1$ when $y = 3$, express x as a function of y. [6]

6. The population of an island at time t is p. The number of birth per unit time is b, and the number of death per unit time is proportional to the population at that time. By taking p as a continuous variable, show that

$\frac{dp}{dt} = b - kp$, where k is a positive constant. [3]

Solve the differential equation with the condition that $p = \frac{b}{5k}$ when $t = 0$.

Express p as a function of t. [4]

Find the time when the population of the island is $\frac{9b}{10k}$. [2]

Sketch the graph of p versus t. [2]

7. The discrete random variable X denotes the number of 'sixes' showing when two fair dice are thrown.

(i) Tabulate the probability distribution of X. [3]

(ii) Two dice are thrown repeatedly. Find the probability that, in 5 throws, the result X = 2 occurs at least once. [3]

(iii) The dice are thrown n times. Find the least value of n such that P(X = 2 occurs at least once in n throws) > 0.9. [3]

(iv) Use an appropriate approximation to estimate the probability that, in 150 throws, the result X = 2 will occur at least twice. [3]

8. The continuous random variable X has probability density function f given by

$f(x) = \begin{cases} k(x+3) & , -3 \le x \le 3 \\ 0 & , \text{otherwise} \end{cases}$. Show that $k = \frac{1}{18}$. [2]

(i) Find E(X) and Var (X). [4]

(ii) Find the value of q such that $P(X \le q) = \frac{1}{4}$. [3]

(iv) Let $Y = aX + b$, where a and b are constants with $a > 0$. Find the values of a and b for which E(Y) = 0 and Var (Y) = 1. [4]

9. The distance travelled by a newspaper vendor in a week for each weekday (Monday to Friday) has mean 13 km and standard deviation 0.8 km. For Saturday and Sunday, the daily distance travelled has mean 11 km and standard deviation 0.7 km. The distances travelled on different days may be assumed to be independent. If D is the average daily distance travelled by the newspaper vendor in a week, find E(D) and Var (D). [4]

Assuming that D has a normal distribution, find the probability that in a randomly chosen week, the mean daily distance travelled by the newspaper vendor is less than 12 km. [3]

10. Melons are sold by weight at a price of RM 1.50 per kg. The masses of melons are normally distributed with a mean of 0.8 kg and a standard deviation of 0.1 kg. Pumpkins are sold by weight at a price of RM 0.50 per kg. The masses of pumpkins are normally distributed with a mean of 1.2 kg and a standard deviation of 0.2 kg. Find the probability that the total price of 5 randomly chosen melons and 3 randomly chosen pumpkins exceeds RM 8. [5]

11. A random variable X has the probability distribution given in the table :

x	2	3	4	5
$P(X = x)$	p	$\frac{2}{10}$	$\frac{3}{10}$	q

(i) Given that E(X) = 4, find p and q. [4]

(ii) Show that Var (X) = 1. [2]

(iii) Find $E(|X - 4|)$. [2]

(iv) Ten independent observations of X are taken. Find the probability that the value of 3 is obtained at most three times. [3]

12. The data shown in the stemplot are the marks in a statistics course obtained by a group of students at a local institution of higher learning.

3	1	4			
4	0	2	3	7	9
5	1	2	2	6	8
6	1	3	3	4	
7	0	2			
8	2				
9	1				

Key : 3 | 1 means 31

(i) Find the percentage of students who obtain less than 40 marks and the percentage of students who obtain at least 80 marks. [2]

(ii) Find the mean and standard deviation of the student's marks. [4]

(iii) Find the median and semi-interquartile range of the student's marks. [4]

(iv) Construct a box plot for the above data. [4]

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1. $2\log_x 3 - \log_3\sqrt{x} = \frac{3}{2} = 2\left[\frac{1}{\log_3 x}\right] - \frac{1}{2}\log_3 x$

$[u=\log_3 x]$: $\frac{2}{u} - \frac{u}{2} = \frac{3}{2} \Rightarrow u^2+3u-4=0$

$\therefore (u-1)(u+4)=0 \Rightarrow u=1$ or -4

$\therefore \log_3 x = 1$ or $-4 \Rightarrow x = 3$ or $\frac{1}{81}$ ✱

2. Let $k = \frac{\sqrt{2}+ai}{1+\sqrt{2}i}$ $(\in \mathbb{R})$

$\therefore k + k\sqrt{2}i = \sqrt{2} + ai$

[Re]: $k=\sqrt{2}$ ✱ [Im]: $a = k\sqrt{2} = 2$ ✱

3. (i) $R = P^2 - 4P - I = \begin{pmatrix}1&1&2\\1&2&1\\2&1&1\end{pmatrix}^2 - 4\begin{pmatrix}1&1&2\\1&2&1\\2&1&1\end{pmatrix} - I$

$= \begin{pmatrix}6&5&5\\5&6&5\\5&5&6\end{pmatrix} - \begin{pmatrix}4&4&8\\4&8&4\\8&4&4\end{pmatrix} - \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = \begin{pmatrix}1&1&-3\\1&-3&1\\-3&1&1\end{pmatrix}$ ✱

(ii) $PR + 4I = \begin{pmatrix}1&1&2\\1&2&1\\2&1&1\end{pmatrix}\begin{pmatrix}1&1&-3\\1&-3&1\\-3&1&1\end{pmatrix} + 4I$

$= \begin{pmatrix}-4&0&0\\0&-4&0\\0&0&-4\end{pmatrix} + \begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix} = \underline{0}$ ✱

4. Let $u = x^2+1$: $\frac{du}{dx} = 2x$

$\int_0^1 x(x^2+1)^4\,dx = \int_1^2 xu^4\left(\frac{du}{2x}\right)$

$= \frac{1}{2}\int_1^2 u^4\,du = \frac{1}{10}\left[u^5\right]_1^2$

$= \frac{1}{10}[32-1] = 3.1$ ✱

5. $x = e^{2t} - 2t \Rightarrow \frac{dx}{dt} = 2e^{2t} - 2$

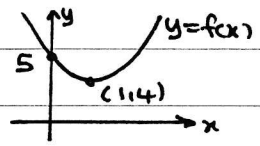
$y = e^t + t \Rightarrow \frac{dy}{dt} = e^t + 1$

$\therefore \frac{dy}{dx} = (e^t+1)/(2e^{2t}-2)$

$t = \ln 2$: $m_t = 2+1/2(2^2)-2 = \frac{1}{2}$ ✱

6(ii) R_f: $\{y : y \geq 4\}$ ✱

(i)



(iii) $f(0)=5$, $f(2)=5$

$\therefore$ f is many-one

$\therefore$ f is not 1-1 $\Rightarrow f^{-1}$ does not exists. ✱

7. $S_n = 1 - \left(\frac{3}{4}\right)^n$: $T_n = S_n - S_{n-1}$

$\therefore T_n = \left[1-\left(\frac{3}{4}\right)^n\right] - \left[1-\left(\frac{3}{4}\right)^{n-1}\right]$

$= \left(\frac{3}{4}\right)^n\left[-1+\frac{4}{3}\right] = \frac{1}{3}\left(\frac{3}{4}\right)^n$

$\therefore r = \frac{T_{n+1}}{T_n} = \frac{1}{3}\left(\frac{3}{4}\right)^{n+1} / \frac{1}{3}\left(\frac{3}{4}\right)^n = \frac{3}{4}$ (constant) ✱

$\therefore S_n$ is a geometric series $(r = \frac{3}{4})$ ✱

$T_1 = S_1 = 1 - \frac{3}{4} = \frac{1}{4}$ ✱

8. (a) $y = \sqrt{\frac{x+1}{x-1}} \Rightarrow y^2 = 1 + \frac{2}{x-1}$

$\therefore 2y\frac{dy}{dx} = -\frac{2}{(x-1)^2} \Rightarrow \frac{dy}{dx} = \frac{-1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}$

$\therefore \frac{dy}{dx} = \frac{-1}{(x-1)\sqrt{x^2-1}}$ ✱

(b) $y^3 - 2xy^2 + 3x^2 - 3 = 0$

$\therefore 3y^2\frac{dy}{dx} - 2\left[y^2 + x\left(2y\frac{dy}{dx}\right)\right] + 6x = 0$

$(2,3)$: $27\frac{dy}{dx} - 18 - 24\frac{dy}{dx} + 12 = 0 \Rightarrow \frac{dy}{dx} = 2$ ✱

9. $(x+y)^{\frac{1}{2}} - (x-y)^{\frac{1}{2}} = \sqrt{x}\left[\left(1+\frac{y}{x}\right)^{\frac{1}{2}} - \left(1-\frac{y}{x}\right)^{\frac{1}{2}}\right]$

$= \sqrt{x}\left[1 + \frac{1}{2}\left(\frac{y}{x}\right) + \frac{1}{2!}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{y}{x}\right)^2 + \frac{1}{3!}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{y}{x}\right)^3 + \ldots - \left(1 + \frac{1}{2}\left(-\frac{y}{x}\right) - \frac{1}{8}\left(-\frac{y}{x}\right)^2 + \frac{1}{16}\left(-\frac{y}{x}\right)^3 + \ldots\right)\right]$

$= \sqrt{x}\left[\frac{y}{x} + \frac{y^3}{8x^3} + \ldots\right] \approx x^{\frac{1}{2}}\left[\frac{y}{x} + \frac{y^3}{8x^3}\right]$ ✱

Let $x+y=6$, $x-y=2$: $x=4$, $y=2$, $\frac{y}{x} = \frac{1}{2}$

$\therefore \sqrt{6} - \sqrt{2} \approx 2\left[\frac{1}{2} + \frac{1}{8}\left(\frac{1}{2}\right)^3\right] = \frac{33}{32}$ ✱

$\sqrt{6}+\sqrt{2} = \sqrt{6}+\sqrt{2} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{6-2}{\sqrt{6}-\sqrt{2}}$

$\approx 4\left[\frac{32}{33}\right] = \frac{128}{33}$ ✱

10. Let $f(x) = 4x^3 - 19x^2 + 11x + 4$

$f(1) = 4 - 19 + 11 + 4 = 0 \Rightarrow (x-1)$ is a factor of $f(x)$ ✱

$f(4) = 256 - 19(16) + 44 + 4 = 0$

$\therefore (x-4)$ is also a factor of $f(x)$

$\therefore f(x) = (x-1)(x-4)(4x+1)$

(ii) $4x^6 - 19x^4 + 11x^2 + 4 = f(x^2)$

For $f(x^2)=0$: $x^2 = 1, 4$ or $-\frac{1}{4}$

$\therefore x = \pm 1, \pm 2$ or $\pm\frac{1}{2}i$

$\therefore$ Real roots $= \pm 1, \pm 2$ ✱

(iii) $f(x) < 20 - 20x$: $(x-1)(x-4)(4x+1) + 20(x-1) < 0$

$\therefore (x-1)[4x^2 - 15x - 4 + 20] < 0$

$\therefore (x-1)\left[x^2 - \frac{15}{4}x + 4\right] < 0$ $\left[\because \left(x-\frac{15}{8}\right)^2 + \frac{31}{64} > 0 \ \forall x \in \mathbb{R}\right]$

$\therefore (x-1)\left[\left(x-\frac{15}{8}\right)^2 + \frac{31}{64}\right] < 0$

$\therefore x - 1 < 0 \Rightarrow x < 1$ $\therefore \{x : x < 1\}$ ✱

11. $y = \frac{\ln x}{x} \Rightarrow xy = \ln x$ $\therefore xy' + y = \frac{1}{x}$

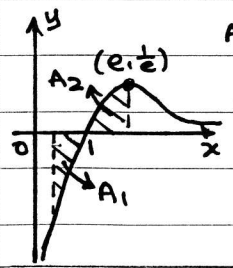
$\therefore [xy'' + y'] + y' = -\frac{1}{x^2}$

For s.point: $y' = 0 \Rightarrow y = \frac{1}{x} = \frac{\ln x}{x}$

$\therefore \ln x = 1 \Rightarrow x = e$ $\therefore (e, \frac{1}{e})$ ✱

$(e, \frac{1}{e})$: $ey'' + 0 + 0 = -\frac{1}{e^2} \Rightarrow y'' = -\frac{1}{e^3}\ (<0)$

$\therefore (e, \frac{1}{e})$ is a max. point. ✱



$A_1 = \int_{\frac{1}{e}}^1 -\frac{\ln x}{x}\,dx$ $\left[\text{let } u = \frac{1}{x} \ \therefore \frac{du}{dx} = -\frac{1}{x^2}\right]$

$= \int_e^1 -\frac{\ln\left(\frac{1}{u}\right)}{1/u}\left(-\frac{du}{u^2}\right)$

$= \int_e^1 \frac{-\ln u}{u}\,du = \int_1^e \frac{\ln u}{u}\,du$

$= A_2$ ✱

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12. $ax^2+bx+c=0 \Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$

$\therefore (x+\frac{b}{2a})^2-(\frac{b}{2a})^2+\frac{c}{a}=0$

$\therefore x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$\therefore x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (roots) ※

If $b^2-4ac \geq 0$: $\sqrt{b^2-4ac} = k\ (\geq 0)$

$\therefore x = \frac{-b\pm k}{2a} \Rightarrow$ roots are real. ※

If $b^2-4ac<0$: $\sqrt{b^2-4ac} = mi$

$\therefore x = \frac{-b\pm mi}{2a} \Rightarrow$ roots are complex ※

$x^2-(k-3)x+k^2+2k+5=0$

For real roots: $b^2-4ac \geq 0$

$(k-3)^2-4(k^2+2k+5) \geq 0 \Rightarrow 3k^2+14k+11 \leq 0$

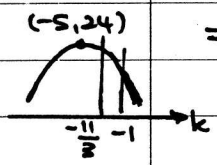
$(3k+11)(k+1) \leq 0$

$\therefore \{k: -\frac{11}{3} \leq k \leq -1\}$ ※

α, β (roots): $\alpha\beta = k^2+2k+5$, $\alpha+\beta = k-3$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = (k-3)^2-2(k^2+2k+5)$

$= -[k^2+10k]-1 = -(k+5)^2+24$ ※



$(\alpha^2+\beta^2)_{max} = -[-\frac{11}{3}+5]^2+24$

$= 22\frac{2}{9}$ ※

1. (i) $\tan(\theta+A)=4$: $\frac{\tan\theta+2}{1-2\tan\theta}=4$

$\therefore \tan\theta = \frac{2}{9}$ ※

(ii) $\sin(\phi+A)=3\cos(\phi-A)$

$\therefore \sin\phi\cos A+\cos\phi\sin A = 3[\cos\phi\cos A+\sin\phi\sin A]$

$\therefore \tan\phi+2 = 3[1+2\tan\phi] \Rightarrow \tan\phi = -\frac{1}{5}$ ※

2. $\sin 4\theta-\sin 2\theta = \cos 3\theta = 2\cos 3\theta\sin\theta$

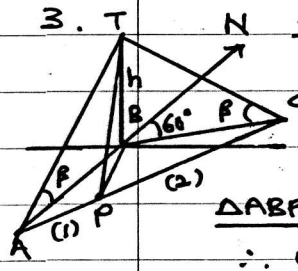
$\therefore \cos 3\theta[2\sin\theta-1]=0$

$\therefore \cos 3\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\therefore 3\theta = \pm\frac{5\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{\pi}{2}$ | $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore \theta = \pm\frac{\pi}{6}, \pm\frac{\pi}{2}, \pm\frac{5\pi}{6}$ ※

3.



ΔBCT: $BC = h\cot\beta$ } $AB=BC=t$

ΔABT: $AB = h\cot\beta$

ΔABC: $\angle BAC = \angle BCA = 30°$

$AP = \frac{1}{3}AC = \frac{1}{3}(2)AB\cos 30°$

$\therefore AP = \frac{1}{\sqrt{3}}t$

ΔABP: $BP^2 = \frac{1}{3}t+t^2-2(\frac{1}{\sqrt{3}}t)t(\frac{\sqrt{3}}{2})$

$\therefore BP = \frac{1}{\sqrt{3}}t$

ΔPBT: (let $\alpha = \angle BPT$) $\tan\alpha = \frac{h}{\frac{1}{\sqrt{3}}t}$

$\therefore \alpha = \tan^{-1}[\sqrt{3}\frac{h}{h\cot\beta}] = \tan^{-1}[\sqrt{3}\tan\beta]$ ※

4. (i) No. of ways $= {}^8C_3 = 56$ ※

(ii) No. of ways $= {}^7C_1(3) = 21$ ※

5. $x^2y^2\frac{dy}{dx} = y^2-4 \Rightarrow \int\frac{y^2}{y^2-4}dy = \int\frac{1}{x^2}dx$

$\therefore \int[1+\frac{1}{y-2}-\frac{1}{y+2}]dy = -\frac{1}{x}+c$

$\therefore y+\ln(y-2)-\ln(y+2) = -\frac{1}{x}+c$ — ①

$x=1, y=3$: $3+0-\ln 5 = -1+c$ — ②

② − ①: $\frac{1}{x} = 4-y+\ln[\frac{y+2}{5(y-2)}]$

$\therefore x = 1/[4-y+\ln(\frac{y+2}{5(y-2)})]$ ※

6. Birth rate: $\frac{dP}{dt} = b$

Death rate: $-\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = -kP$ $(k>0)$

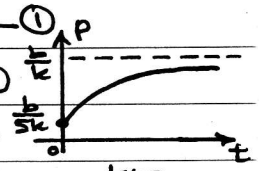
$\therefore$ Total rate: $\frac{dP}{dt} = b-kP$ ※

$\int\frac{-k}{b-kP}dP = \int -k\,dt$

$\therefore \ln(b-kP) = -kt+c$ — ①

$t=0, P=\frac{b}{5k}$: $\ln\frac{4}{5}b = c$ — ②

① − ②: $\ln[\frac{5(b-kP)}{4b}] = -kt$



$\therefore \frac{5(b-kP)}{4b} = e^{-kt} \Rightarrow P = \frac{b}{5k}[5-4e^{-kt}]$ ※

$P = \frac{9b}{10k}$: $t = -\frac{1}{k}\ln[5(\frac{b}{10})/4b] = \frac{1}{k}\ln 8 = \frac{3}{k}\ln 2$ ※

7. (i) $X = 0, 1, 2$: $P(X=0) = (\frac{5}{6})^2 = \frac{25}{36}$

$P(X=1) = (\frac{5}{6})(\frac{1}{6})(2) = \frac{10}{36}$

$P(X=2) = (\frac{1}{6})^2 = \frac{1}{36}$

x	0	1	2
$P(X=x)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

※

(ii) Let Y = no. of $X=2$, $Y \sim B(5, \frac{1}{36})$

$P(Y \geq 1) = 1-(\frac{35}{36})^5 = 0.1314$ ※

(iii) Let $W \sim B(n, \frac{1}{36})$: For $P(W \geq 1) > 0.9$

$\therefore 1-(\frac{35}{36})^n > 0.9 \Rightarrow n\lg(\frac{35}{36}) < -1$

$\therefore n > 81.74 \Rightarrow n_{min} = 82$ ※ $(\because n \in \mathbb{N})$

(iv) Let $T \sim B(150, \frac{1}{36}) \rightarrow T \sim P_0(\frac{25}{6})$

$P(T \geq 2) = 1-(P_0+P_1) = 1-e^{-\frac{25}{6}}[1+\frac{25}{6}]$

$= 0.9199$ ※

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8. $\int_{-\infty}^{\infty} f(x)\,dx = 1$: $\int_{-3}^{3} k(x+3)\,dx = 1$

$\therefore \frac{k}{2}\left[(x+3)^2\right]_{-3}^{3} = 1 = \frac{k}{2}[36-0] \Rightarrow k = \frac{1}{18}$ #

(i) $E(X) = \int_{-3}^{3} \frac{1}{18}x(x+3)\,dx = \frac{1}{18}\left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_{-3}^{3}$

$= \frac{1}{18}\left[\frac{27+27}{3} + \frac{3}{2}[9-9]\right] = 1$ #

$E(X^2) = \int_{-3}^{3} \frac{1}{18}x^2(x+3)\,dx = \frac{1}{18}\left[\frac{x^4}{4} + x^3\right]_{-3}^{3}$

$= \frac{1}{18}\left[\frac{81-81}{4} + (27+27)\right] = 3$

$\therefore Var(X) = 3 - 1^2 = 2$ #

(ii) $P(X \le q) = \frac{1}{4}$: $\int_{-3}^{q} \frac{1}{18}(x+3)\,dx = \frac{1}{4}$

$\therefore \left[(x+3)^2\right]_{-3}^{q} = 9 = (q+3)^2 - 0$

$\therefore q+3 = \pm 3 \Rightarrow q = 0$ # $[\because -3 < q < 3]$

(iv) $E(Y) = 0$: $a(1) + b = 0 \Rightarrow b = -a$

$Var(Y) = 1$: $a^2(2) = 1 \Rightarrow a = \frac{1}{\sqrt{2}}$ # $(\because a > 0)$

$\therefore b = -\frac{1}{\sqrt{2}}$ #

9. Let X = distance travelled each day (M→F)

Y = distance travelled in a day (Sat–Sun)

$D = \frac{1}{7}(X_1 + X_2 + X_3 + X_4 + X_5 + Y_1 + Y_2)$

$E(D) = \frac{1}{7}[5(13) + 2(11)] = \frac{87}{7}$ #

$Var(D) = \left(\frac{1}{7}\right)^2[5(0.8)^2 + 2(0.7)^2] = \frac{209}{2450}$ #

$P(D < 12) = P\left(Z < \frac{12 - \frac{87}{7}}{\sqrt{\frac{209}{2450}}}\right)$

$= P(Z < -1.467) = 0.0712$ #

10. M = mass of melon $M \sim N(0.8, 0.1^2)$

P = mass of pumpkin $P \sim N(1.2, 0.2^2)$

Let $T = 1.5\sum_{1}^{5} M_i + 0.5\sum_{i=1}^{3} P_i$

$E(T) = 1.5[5(0.8)] + 0.5[3(1.2)] = 7.8$

$Var(T) = 1.5^2[5(0.1)^2] + 0.5^2[3(0.2)^2] = 0.1425$

$\therefore T \sim N(7.8, 0.1425)$

$P(T > 8) = P\left(Z > \frac{8 - 7.8}{\sqrt{0.1425}}\right)$

$= P(Z > 0.5298) = 0.2981$ #

11. (i) $\sum P(X=x) = 1$: $p + q + \frac{2}{10} + \frac{3}{10} = 1$

$\therefore p + q = \frac{1}{2}$ — ① $\times 2 \Rightarrow 2p + 2q = 1$

$E(X) = 4$: $2(p) + 3\left(\frac{2}{10}\right) + 4\left(\frac{3}{10}\right) + 5q = 4$

$\therefore 2p + 5q = 2.2$ — ②

② − ① : $3q = 1.2 \Rightarrow q = 0.4$ # $\therefore p = 0.1$ #

(ii) $E(X^2) = 2^2(0.1) + 3^2(0.2) + 4^2(0.3) + 5^2(0.4)$

$= 17$

$\therefore Var(X) = 17 - 4^2 = 1$ #

(iii) $E(|X-4|) = 2(0.1) + 1(0.2) + 0 + 1(0.4)$

$= 0.8$ #

(iv) Let Y = no. of 3 obtained, $Y \sim B(10, 0.2)$

$P(Y \le 3) = 0.8^{10} + 10(0.8)^9(0.2) + 45(0.8)^8(0.2)^2$

$+ 120(0.8)^7(0.2)^3 = 0.8791$ #

12. (i) Percentage $(<40) = \frac{2}{20} \times 100\% = 10\%$ #

Percentage $(\ge 80) = \frac{2}{20} \times 100\% = 10\%$ #

(ii) $n = 20$, $\sum x = 1121$, $\sum x^2 = 67293$

$\bar{x} = \frac{1121}{20} = 56.05$ #

$s = \sqrt{\frac{67293}{20} - 56.05^2} = 14.93$ #

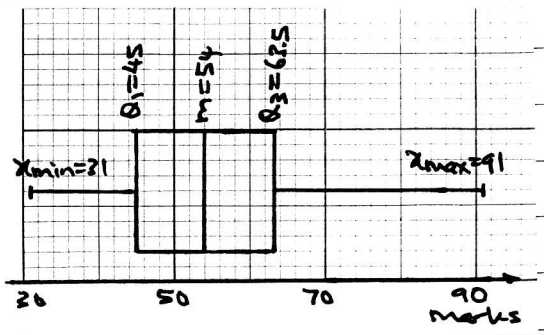
(iii) $m = \frac{1}{2}[52 + 56] = 54$ #

$Q_1 = \frac{1}{2}[43 + 47] = 45$

$Q_3 = \frac{1}{2}[63 + 64] = 63.5$

$SIQR = \frac{1}{2}[63.5 - 45] = 9.25$ #

(iv)



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