SMK ST GEORGE (2005)
MATHEMATICS T (PAPER 1)

1. Solve the equation $\log_{\frac{1}{4}} x + 2\log_{16} x^2 - \frac{3}{2} = 0$. [4]

2. Find the range of x where $|x^2 - 4| < 3x$. [5]

3. Evaluate $\int_0^{\pi} x^2 \sin x \, dx$. [5]

4. Find the equation of tangent to the curve $y = e^x$ at the point T where $x = t$.
The tangent at T intersects the x–axis at U. The normal to the curve at the point T intersects the y–axis at V. Show that the area of triangle TUV is $\frac{1}{2}e^t(e^{2t} + 1)$. [6]

5. Given that $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$. Show that $\mathbf{A}^3 = \mathbf{A}$ and thus deduce the matrix of $\mathbf{A}^{40}$. [6]

6. Given that $1 + 2i$ is one of the roots of the equation $x^4 - 3x^3 + 8x^2 - 7x + 5 = 0$, find the remaining roots. [7]

7. (a) Find the value of the term independent of x in the expansion of $(3x^2 - \frac{1}{2x})^9$.

   (b) Use the binomial theorem to find the value of the fifth root of 3127 correct to 4 decimal places. [8]

8. Sketch the curves with equation $y = 4x - x^2$ and $y = \frac{4}{x} - 1$. Find the coordinates of the points of intersection of the curves.
Show that the area of the region enclosed by the curves is $12 - 8\ln 2$. [8]

9. If $y = (x^2 + 4)e^{-\frac{1}{2}x}$, show that $\frac{dy}{dx} \le 0$ for all real values of x. [4]
Find the values of x for which (a) $\frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} = 0$. [5]
Sketch the curve $y = (x^2 + 4)e^{-\frac{1}{2}x}$, showing clearly the behaviour of the curve at the points where $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$. [4]

10. Two function f and g each has domain D given by $\{x \in \mathcal{R} : x > 1\}$.
The functions are defined by $f : x \to \log_{10}(x + 1)$ and $g : x \to x^2 + 2x$.

    (a) State the range of f and define f^{-1}. [3]

    (b) Determine whether g is one-to-one, giving a reason for your answer, and state the range of g. [3]

    (c) Define the composite function $f \circ g$ and state its range. [4]

    (d) Find the value of x such that $f^{-1}g(x) = 10^3 - 1$. [3]

11. Prove that the equation of the chord joining $P(cp, \frac{c}{p})$ and $Q(cq, \frac{c}{q})$ on the rectangular hyperbola $xy = c^2$ is given by $pqy + x = c(p + q)$. [3]
Given that PQ subtends a right angle at the point $R(cr, \frac{c}{r})$ on the curve, prove that

    (a) PQ is parallel to the normal at R on the curve. [4]

    (b) the midpoint of PQ lies on the line $y + r^2x = 0$. [4]

12. Express $u_r = \frac{1}{r^2 + 2r}$ in partial fractions. [3]
Using the result obtained,

    (i) show that $u_r^2 = \frac{1}{4}\left(\frac{1}{r^2} + \frac{1}{r+2} - \frac{1}{r} + \frac{1}{(r+2)^2}\right)$. [2]

    (ii) show that $\sum_{r=1}^{n} u_r = \frac{1}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$, and

    determine the value of $\sum_{r=1}^{\infty} u_r$ and $\sum_{r=1}^{\infty}(u_{r+2} + \frac{1}{4^r})$. [9]

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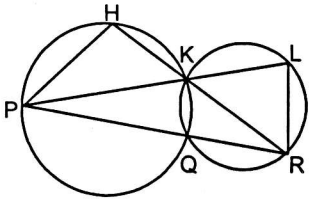
SMK ST GEORGE (2005)
MATHEMATICS T (PAPER 2)

1. Find all values of x, where $0^\circ < x < 360^\circ$, which satisfy the equation
$3\cos 2x - \sin x = 2$. [5]

2. Three forces $2\underline{i} - a\underline{j}$, $b\underline{i} - 7\underline{j}$ and $4\underline{i} - 3\underline{j}$ act on a point A. Determine the values of a and b if the resultant force on A is $3\underline{i} - 5\underline{j}$. [4]

3. Prove the following identities :
(a) $\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta - \frac{2\pi}{3}\right) = 0$. [4]
(b) $\cos^2\theta + \cos^2\left(\theta + \frac{2\pi}{3}\right) + \cos^2\left(\theta - \frac{2\pi}{3}\right) = \frac{3}{2}$. [4]

4. In a flu epidemic, the fraction of the population in a certain country that was infected was represented by x. The rate of spread of the disease was defined as the rate of change of x with respect to t, where t was the time in days, after the disease was first detected. Given that the rate of spread of the disease was directly proportional to the product of x and $(1 - x)$, write a differential equation that connects the variables x and t. The disease was first detected after $\frac{1}{8}$ of the population was infected. If the epidemic persisted, $\frac{1}{2}$ of the population would be infected 10 days after the disease was first detected. Find the percentage of the population that would be infected 20 days after the disease was first detected. [11]

5. 

In the diagram, PQR, PKL and HKR are straight lines.
(i) Show that PHK and KLR are supplementary.
(ii) If a circle can be drawn passing through the points P, R, L and H, show that its diameter is PR. [10]

6. A ship A is sailing on a bearing of 200° at a speed of 12 knot. A ship B is sailing on a bearing of 320°. When ship A is 1 nautical mile north of ship B, the captains of both ships discover that their ships are in the path of collision. Find the speed of ship B.
If at that point, without changing its direction, ship B reduces its speed to half of its original value, find the closest distance between ship A and ship B. [11]

7. Events A and B are such that $P(A) = 0.2$, $P(B' \mid A) = 0.9$ and $P(B \mid A') = 0.5$. Find $P(A \cap B)$ and $P(A \cup B)$. [7]

8. A discrete random variable X has the probability function
$$P(X = x) = \begin{cases} \frac{x+1}{10} & , x = 0,1,2 \\ \frac{9-2x}{10} & , x = 3,2 \\ 0 & , \text{otherwise} \end{cases}$$
Find E(X) and Var(X). [6]

9. The probability that a visitor to a computer shop will buy a computer is 0.2. If six people visit the shop at a certain time, find the probability that 4 or more visitors will buy a computer. [4]

10. If $X \sim N(100, 64)$ and $P(X > h) = 2P(X \le h)$, find the value of h. [6]

11. The continuous random variable X has the probability density function $f(x)$ given by
$$f(x) = \begin{cases} \frac{2x}{3} & , 0 < x \le 1 \\ 1 - \frac{x}{3} & , 1 < x \le 3 \\ 0 & , \text{otherwise} \end{cases}$$
(i) Find the cumulative distribution function of X.
(ii) Calculate the median of X.
(iii) Calculate $P(0.5 < X \le 2)$. [12]

12. The daily highest temperature (in Celsius) for a certain month in town A is listed as follows :

Temperature (°C)	≤ 25	≤ 27	≤ 29	≤ 31	≤ 33	≤ 37
Cumulative frequency (days)	0	2	7	16	24	30

(i) Draw a histogram for the data above. [3]
(ii) Find the mean, median and standard deviation in Celsius. [10]
(iii) If the relation between Celsius scale (°C) and the Fahrenheit scale (°F) is given by $F = \frac{9}{5}C + 32$, find the mean temperature in Fahrenheit. [1]

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SMK St. George (2005)

No:..... MT1 Date:.....

1. $\log_{\frac{1}{4}} x + 2\log_{16} x^2 - \frac{3}{2} = 0$ $[\text{let } \log_2 x = u]$

$\therefore \frac{u}{-2} + 4\left[\frac{u}{4}\right] - \frac{3}{2} = 0 \Rightarrow \log_2 x = 3$

$\therefore x = 2^3 = 8$ ✱

2. $|x^2-4| < 3x$

$\therefore -3x < x^2-4$ and $x^2-4 < 3x$

$\therefore x^2+3x-4>0$ | $x^2-3x-4<0$

$\therefore (x+4)(x-1)>0$ | $(x-4)(x+1)<0$

$x<-4$ or $1<x$ | $-1<x<4$

$\therefore \{x : 1<x<4\}$ ✱

3. $\int_0^\pi x^2 \sin x\,dx = \left(-x^2\cos x\right]_0^\pi + \int_0^\pi 2x\cos x\,dx$

$= -\pi^2(-1) - 0 + \left[2x\sin x\right]_0^\pi - \int_0^\pi 2\sin x\,dx$

$= \pi^2 + [0+0] + 2\left(\cos x\right]_0^\pi$

$= \pi^2 + 2(-1-1) = \pi^2 - 4$ ✱

4. $y = e^x \Rightarrow y' = e^x$

$T(t, e^t)$: $m = e^t$

$\therefore$ tangent(T): $y - e^t = e^t(x-t) \Rightarrow y = e^t(x-t+1)$ ✱

For U: $(y=0) \Rightarrow x = t-1$ $\therefore U(t-1, 0)$

Let $V(V, 0)$: $m_{TV} = -e^{-t} = \frac{-e^t}{V-t}$

$\therefore V = e^{2t}+t$ $\therefore V(e^{2t}+t, 0)$

$\therefore$ Area $= \frac{1}{2}\left[(e^{2t}+t)-(t-1)\right]\left[e^t\right]$

$= \frac{1}{2}e^t\left(e^{2t}+1\right)$ ✱

5. $A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & 1 \\ 4 & -2 & 2 \end{bmatrix}$

$\therefore A^3 = A^2 . A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} = A$ ✱

$A^{40} = (A^3)^{13} A = A^{14} = (A^3)^4 A^2 = A^6$

$= (A^3)^2 = A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & 1 \\ 4 & -2 & 2 \end{bmatrix}$ ✱

6. $(1+2i)$ is a root for $f(x) = x^4 - 3x^3 + 8x^2 - 7x + 5 = 0$

$\therefore (1-2i)$ is also a root

$\therefore x^2 - 2x + 5$ is a factor of $f(x)$

$\therefore f(x) = x^4 - 3x^3 + 8x^2 - 7x + 5$

$= (x^2-2x+5)(x^2-x+1)$

For $f(x)=0$: $x = 1 \pm 2i,\ \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$\therefore$ Remaining roots $= 1-2i,\ \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ ✱

7(a) $\left(3x^2 - \frac{1}{2x}\right)^9 = \sum_{r=0}^{9} {}^9C_r (3x^2)^{9-r}\left(-\frac{1}{2x}\right)^r$

For x^0: $(r=6)$ term $= {}^9C_6 (3x^2)^3 \left(-\frac{1}{2x}\right)^6$

$= \frac{567}{16}$ ✱

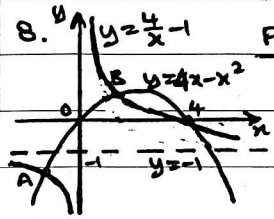
(b) Let $f(x) = \sqrt[5]{1+2x} = 1 + \frac{1}{5}(2x) + \frac{\frac{1}{5}(-\frac{4}{5})}{2!}(2x)^2$

$+ \frac{\frac{1}{5}(-\frac{4}{5})(-\frac{9}{5})}{3!}(2x)^3 + \ldots = 1 + \frac{2}{5}x - \frac{8}{25}x^2 + \frac{48}{125}x^2 + \ldots$

$\therefore f\left(\frac{1}{3125}\right) = \sqrt[5]{\frac{3127}{3125}} = \frac{1}{5}\sqrt[5]{3127}$

$\therefore \sqrt[5]{3127} = 5\left[1 + \frac{2}{5}\left(\frac{1}{3125}\right) - \frac{8}{25}\left(\frac{1}{3125}\right)^2 + \frac{48}{125}\left(\frac{1}{3125}\right)^2 + \ldots\right]$

$= 5.0006$ (4 d.p) ✱

8. 

For A, B, C: $\frac{4}{x} - 1 = 4x - x^2$

$\therefore 4 - x = x^2(4-x)$

$\therefore (4-x)(x^2-1) = 0$

$\therefore x = 4$ or ± 1

$\therefore A(-1,-5)$, $B(1,3)$, $C(4,0)$ ✱

Area $= \int_1^4 \left[4x - x^2 - \frac{4}{x} + 1\right]dx$

$= \left[2x^2 - \frac{x^3}{3} - 4\ln x + x\right]_1^4$

$= 2(16-1) - \frac{1}{3}(64-1) - 4\ln 4 + (4-1) = 12 - 8\ln 2$ ✱

9. $y = (x^2+4)e^{-\frac{1}{2}x} \Rightarrow y' = 2xe^{-\frac{1}{2}x} + (x^2+4)\left(-\frac{1}{2}e^{-\frac{1}{2}x}\right)$

$\therefore y' = \frac{1}{2}e^{-\frac{1}{2}x}\left[-(x-2)^2\right]$ $(\forall x \in \mathbb{R})$

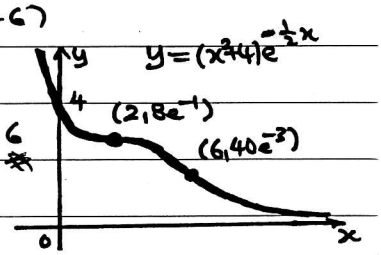
$\left[\because e^{-\frac{1}{2}x} > 0,\ -(x-2)^2 \le 0,\ \forall x \in \mathbb{R}\right]$: $\frac{dy}{dx} \le 0$ ✱

$y'' = -\frac{1}{4}e^{-\frac{1}{2}x}\left[-(x-2)^2\right] + \frac{1}{2}e^{-\frac{1}{2}x}\left[-2(x-2)\right]$

$= \frac{1}{4}e^{-\frac{1}{2}x}(x-2)(x-6)$

(a) $\frac{dy}{dx} = 0$: $x = 2$ ✱

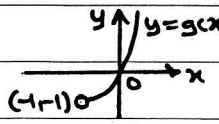
(b) $\frac{d^2y}{dx^2} = 0$: $x = 2$ or 6 ✱



10. (a) R_f: $\{y : -\infty < y < \infty\} \equiv \mathbb{R}$ ✱

$f^{-1} : x \to 10^x - 1$, $x \in \mathbb{R}$. ✱

(b) $g(x) = x^2 + 2x = (x+1)^2 - 1$



The line $y=k$ does not intersects the graph of g at more than one point. $\Rightarrow$ g is one-one ✱

R_g: $\{y : y > -1\}$ ✱

(c) $fg(x) = f(x^2+2x) = \lg(x^2+2x+1) = 2\lg(x+1)$

$\therefore fg : x \to 2\lg(x+1)$, $x > -1$ ✱

R_{fg}: $\{y : -\infty < y < \infty\} \equiv \mathbb{R}$ ✱

(d) $f^{-1}g(x) = 10^3 - 1 \Rightarrow x^2 + 2x = f(10^2-1) = 3$

$\therefore x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0$

$[\because x > -1]$: $x = 1$ ✱

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Uni

11. $m_{PQ} = \dfrac{\frac{c}{p}-\frac{c}{q}}{cp-cq} = \dfrac{c(q-p)}{pq} \Big/ c(p-q) = -\dfrac{1}{pq}$

$\therefore$ PQ: $y - \frac{c}{p} = -\frac{1}{pq}(x-cp)$

$\therefore\ pqy + x = c(p+q)$ ✱

(a) $\angle PRQ = 90^\circ$: $m_{PR}\cdot m_{QR} = -1 = \left(-\frac{1}{pr}\right)\left(-\frac{1}{qr}\right)$

$\therefore\ r^2 = -\frac{1}{pq} \Rightarrow m_{N(R)} = m_{PQ}$

$\therefore$ PQ // Normal at R ✱ $\left[m_{N(R)} = -\frac{1}{r^2}\right]$

(b) $M_{PQ} = (x,y) = \left(\frac{cp+cq}{2},\ \frac{\frac{c}{p}+\frac{c}{q}}{2}\right)$

$\therefore\ 2x = c(p+q)$ and $2y = \frac{c(p+q)}{pq}$

$\therefore\ \frac{x}{y} = pq = -\frac{1}{r^2} \Rightarrow y + r^2x = 0$ [locus of M]

$\therefore$ M lies on the line $y + r^2x = 0$ ✱

12. $u_r = \frac{1}{r^2+2r} = \frac{A}{r} + \frac{B}{r+2} = \frac{A(r+2)+Br}{r^2+2r}$

$\therefore\ 1 = A(r+2) + Br$

$r=0$: $A = \frac{1}{2}$, $r=-2$: $B = -\frac{1}{2}$

$\therefore\ u_r = \frac{1}{2}\left[\frac{1}{r} - \frac{1}{r+2}\right]$ ✱

(i) $(u_r)^2 = \frac{1}{4}\left[\frac{1}{r^2} + \frac{1}{(r+2)^2} - \frac{2}{r(r+2)}\right]$

$= \frac{1}{4}\left[\frac{1}{r^2} + \frac{1}{(r+2)^2} - \frac{1}{r} + \frac{1}{r+2}\right]$ ✱

(ii) $\sum_{r=1}^{n} u_r = \frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+2}\right)$

$= \frac{1}{2}\left[\left(1+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{n}\right) - \left(\frac{1}{3}+\dots+\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}\right)\right]$

$= \frac{1}{2}\left[\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$ ✱

$\sum_{r=1}^{\infty} u_r = \lim_{n\to\infty}\left[\sum_{r=1}^{n} u_r\right] = \lim_{n\to\infty}\left[\frac{1}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)\right]$

$= \frac{1}{2}\left[\frac{3}{2} - 0 - 0\right] = \frac{3}{4}$

$\sum_{r=1}^{\infty}\left(u_{r+2} + \frac{1}{4^r}\right) = (u_3+u_4+\dots) +$

$\left(\frac{1}{4}+\frac{1}{4^2}+\dots\right) = \sum_{r=1}^{\infty} u_r - (u_1+u_2) + \frac{\frac{1}{4}}{1-\frac{1}{4}}$

$= \frac{3}{4} - \left[\frac{1}{1+2} + \frac{1}{4+4}\right] + \frac{1}{3} = \frac{5}{8}$ ✱

1. $3\cos 2x - \sin x = 2 = 3(1-2s^2) - s$

$\therefore\ 6s^2 + s - 1 = 0 \Rightarrow (3s-1)(2s+1) = 0$

$\therefore\ \sin x = \frac{1}{3}$ or $-\frac{1}{2}$

$\therefore\ x = 19.5^\circ, 160.5^\circ, 210^\circ, 330^\circ$ ✱

2. $F_R = F_1 + F_2 + F_3$:

$\begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}2\\a\end{pmatrix} + \begin{pmatrix}b\\-7\end{pmatrix} + \begin{pmatrix}4\\-8\end{pmatrix} \Rightarrow a = 5,\ b = -3$ ✱

3.(a) $\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta - \frac{2\pi}{3}\right)$

$= \cos\theta + \cos\theta\left(-\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta\left(-\frac{1}{2}\right) + \sin\theta\left(\frac{\sqrt{3}}{2}\right) = 0$ ✱

(b) $\cos^2\theta + \cos^2\left(\theta + \frac{2\pi}{3}\right) + \cos^2\left(\theta - \frac{2\pi}{3}\right)$

$= c^2 + \frac{1}{4}\left(-c - \sqrt{3}s\right)^2 + \frac{1}{4}\left(-c + \sqrt{3}s\right)^2$

$= c^2 + \frac{1}{4}\left(c^2 + 2\sqrt{3}cs + 3s^2\right) + \frac{1}{4}\left(c^2 - 2\sqrt{3}cs + 3s^2\right)$

$= \frac{3}{2}\left(c^2+s^2\right) = \frac{3}{2}(1) = \frac{3}{2}$ ✱

4. $\frac{dx}{dt} \propto x(1-x) \Rightarrow \frac{dx}{dt} = kx(1-x)$ ✱

$\int_{\frac{1}{8}}^{x} \frac{dx}{x(1-x)} = \int_0^t k\,dt = \int_{\frac{1}{8}}^{x}\left[\frac{1}{x} + \frac{1}{1-x}\right]dx$

$\therefore\ k[t]_0^t = [\ln x - \ln(1-x)]_{1/8}^{x}$

$\therefore\ kt = \ln\left(\frac{x}{1-x}\right) + \ln 7 = \ln\left(\frac{7x}{1-x}\right)$

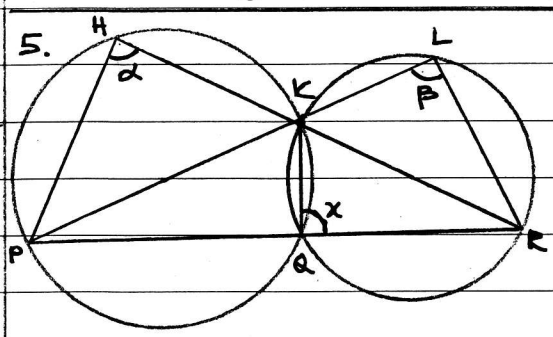
$t=10, x=\frac{1}{2}$: $10k = \ln 7 \Rightarrow k = \frac{1}{10}\ln 7$

$t=20$: $20\left(\frac{1}{10}\ln 7\right) = \ln\left(\frac{7x}{1-x}\right)$

$\therefore\ 49 = \frac{7x}{1-x} \Rightarrow x = \frac{7}{8}$

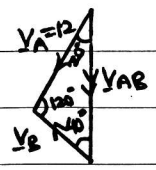
$\therefore$ Percentage infected $(t=20) = 87.5\%$ ✱

5.



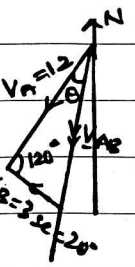
Statements	Reasons
(i) $\alpha = x$	Exterior angle of cyclic PQKH.
$x + \beta = 180^\circ$	Opposite angles of cyclic QRLK.
$\therefore\ \alpha + \beta = 180^\circ$	
$\therefore\ \angle PHK$ and $\angle KLR$ are supplementary ✱	
(ii) $\alpha = \beta$	Angles in the same segment [Given PRLH are concyclic]
$\therefore\ \alpha + \alpha = 180^\circ$	$\alpha + \beta = 180^\circ$
$\therefore\ \alpha = 90^\circ = \beta$	
$\therefore$ PR is diameter of circle passing through PRLH. ✱	Angle in semicircle $= 90^\circ$.

6. Collision: $\underline{V}_{AB}$: due South



$\frac{|\underline{V}_B|}{\sin 20^\circ} = \frac{|\underline{V}_A|}{\sin 40^\circ} \Rightarrow |\underline{V}_B| = 12\,\frac{\sin 20^\circ}{2\sin 20^\circ\cos 20^\circ}$

$\therefore\ |\underline{V}_B| = 6\sec 20^\circ = 6.385$ knot ✱



$|\underline{V}_{AB}|^2 = 12^2 + (3\sec 20^\circ)^2 - 2(12)(3\sec 20^\circ)\left(-\frac{1}{2}\right)$

$\therefore\ |\underline{V}_{AB}| = 13.87$ knot

$\frac{\sin\theta}{3\sec 20^\circ} = \frac{\sin 120^\circ}{13.87} \Rightarrow \theta = 11.5^\circ$

$\therefore$ Shortest distance $= 1(\sin 8.5^\circ)$

$= 0.148$ nautical miles. ✱

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7. $P(B'|A) = 0.9 = \dfrac{P(B'\cap A)}{P(A)}$

$\therefore P(A) - P(A\cap B) = 0.9(0.2)$

$\therefore P(A\cap B) = 0.2 - 0.18 = 0.02$ #

$P(B|A') = 0.5 = \dfrac{P(B\cap A')}{1-P(A)} \Rightarrow P(B\cap A') = 0.4$

$\therefore P(A\cup B) = P(A) + P(B\cap A') = 0.6$ #

8.

x	0	1	2	3	4
$P(X=x)$	0.1	0.2	0.3	0.3	0.1

$E(X) = 0 + 0.2 + 0.6 + 0.9 + 0.4 = 2.1$ #

$E(X^2) = 0 + 0.2 + 1.2 + 2.7 + 1.6 = 5.7$

$\therefore Var(X) = 5.7 - 2.1^2 = 1.29$ #

9. Let X = no. of visitors buy computer, $X \sim B(6, 0.2)$

$P(X \geq 4) = 15(0.2)^4(0.8)^2 + 6(0.2)^5(0.8)$

$+ (0.2)^6 = 0.01696$ #

10. $X \sim N(100, 64)$:

$P(X > h) = 2P(X \leq h) = 2[1 - P(X > h)]$

$\therefore P(X > h) = \frac{2}{3} = P(Z > \frac{h-100}{8})$

$\therefore \frac{h-100}{8} = -Z_{\frac{1}{3}} = -0.431 \Rightarrow h = 96.552$ #

11. (i) $0 < x \leq 1$: $P(X \leq x) = \int_0^x \frac{2}{3}x\,dx$

$= [\frac{1}{3}x^2]_0^x = \frac{1}{3}x^2$

$1 < x \leq 3$: $P(X \leq x) = \frac{1}{3}(1)^2 + \int_1^x [1 - \frac{1}{3}x]\,dx$

$= \frac{1}{3} + [x - \frac{1}{6}x^2]_1^x = \frac{1}{3} + [x - \frac{1}{6}x^2 - 1 + \frac{1}{6}]$

$= \frac{1}{6}(6x - 3 - x^2)$

$\therefore F(x) = P(X \leq x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{3}x^2 & 0 < x \leq 1 \\ \frac{1}{6}(6x-3-x^2) & 1 < x \leq 3 \\ 1 & 3 < x \end{cases}$ #

(ii) $P(X < m) = \frac{1}{2}$:

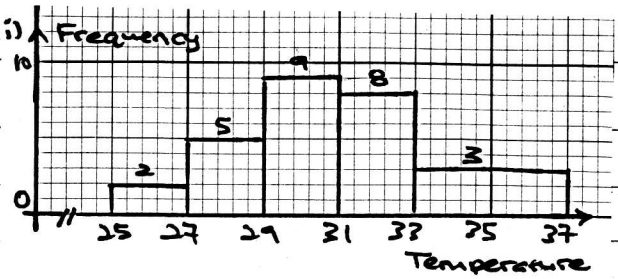
$\frac{1}{6}(6m - 3 - m^2) = \frac{1}{2} \Rightarrow m^2 - 6m + 6 = 0$

$\therefore m = \dfrac{6 - \sqrt{36-24}}{2} = 3 - \sqrt{3}$ ($\because 1 < m \leq 3$) #

(iii) $P(0.5 < X \leq 2) = P(X \leq 2) - P(X \leq 0.5)$

$= \frac{1}{6}[12 - 3 - 4] - \frac{1}{3}(\frac{1}{2})^2 = \frac{3}{4}$ #

12. (i)



(ii) $n = 30$, $\Sigma fx = 928$, $\Sigma fx^2 = 28914$

$\therefore \bar{x} = \frac{928}{30} = 30.93$ #

$m = x_{15} = 29 + (\frac{15-7}{9})(2) = 30.78$ #

$s = [\frac{28914}{30} - (\frac{928}{30})^2]^{1/2} = 2.63$ #

(iii) $\text{mean}(F) = \frac{9}{5}(\frac{928}{30}) + 32 = 87.68$ #

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