SMK COCHRANE KL (2005)
MATHEMATICS T (PAPER 1)

1. A curve is defined by the equation $x^3 - y^3 + 3xy - 1 = 0$.
Find the gradient of the curve at the point (2, –1). [4]

2. Show that $(x + 1)$ is the only linear factor of the polynomial $P(x) = x^3 + 3x^2 + 9x + 7$. [5]

3. (a) Express $\dfrac{20\sqrt{3} - 12i}{\sqrt{3} - 9i}$ in the form of $a + bi$, where a and b are real numbers. [2]

(b) Express the value of $\left(\dfrac{20\sqrt{3} - 12i}{\sqrt{3} - 9i}\right)^5$ in the form of $x + yi$,

where x and y are real numbers. [4]

4. (a) Find the set of values of x such that $x^3 > 6x^2 - 9x$. [3]

(b) Sketch the graphs of $y = 5 - x^2$ and $y = |1 - x|$ on the same coordinate axes.
Hence, find the solution set of $5 - x^2 \geq |1 - x|$. [5]

5. A small company producing children's toys plans to increase its output. The number of toys produced is to be increased by 8 each week until the weekly number produced reaches 1000. In week 1, the number to be produced is 280; in week 2, the number is 288; etc. Show that the weekly number produced will be 1000 in week 91. [3]

From week 91 onwards, the number produced each week is to remain at 1000.
Find the total number of toys to be produced over the first 104 week of the plan. [4]

6. The function f is defined by $f(x) = \begin{cases} 2x - \ln 2 - 1 & , x < 2 \\ 2 & , x = 2 \\ 3 - \ln x & , x > 2 \end{cases}$.

(a) Find $\lim\limits_{n \to 2^+} f(x)$ and $\lim\limits_{n \to 2^-} f(x)$.

Hence determine whether f is continuous at $x = 2$. [4]

(b) Sketch the graph of f. [3]

7 (a) Using the substitution $u = \sqrt{x}$, find $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$. [4]

(b) Find the exact value of $\displaystyle\int_0^2 xe^{2x}\,dx$. [4]

8. Given that $A = \begin{pmatrix} -8 & -6 & -6 \\ 2 & 0 & 2 \\ 8 & 8 & 6 \end{pmatrix}$. Find A^2 and hence, deduce A^{-1}. [4]

Express the equations $-8x - 6y - 6z = 13$, $2x + 2z = 7$ and $8x + 8y + 6z = -25$ in matrix equation.

Hence, solve the system of linear equations for the values of x, y and z. [4]

9. Obtain the first three terms in the expansion of $\sqrt{\dfrac{3+x}{3-x}}$, in ascending powers of x,

and state the range of values of x for which the expansion is valid. [6]

By using $x = \frac{1}{5}$, find an approximation for $\sqrt{14}$ in the form $\frac{p}{q}$,

where p and q are integers. [3]

10. Three points have coordinates $A(1, 2)$, $B(4, 6)$ and $C(2, 4)$. The point D is such that $ABCD$ form a parallelogram. Find the coordinates of D. [2]
The line through B perpendicular to AC meets AC produced at E.

(a) Find the coordinates of E. [3]

(b) Find the area of $ABCD$. [2]

(c) Find the equation of the circle passing through the points A, B and E. [4]

11. (a) Using the trapezium rule with 5 ordinates, obtain an approximation of the area enclosed by the curve $y = \dfrac{x}{1+x^2}$, the x–axis and the line $x = 1$.

Give your answer correct to 5 decimal places.

Hence, estimate the value of ln 2, giving your correct to 3 decimal places. [7]

(b) The curve $y = e^{3x+2}$ and the line $3x + 2y = 0$ intersect at a point T.

(i) Verify that T has coordinates $\left(-\frac{2}{3}, 1\right)$. [1]

(ii) Find the volume of the solid obtained by a complete revolution of the area enclosed by curve $y = e^{3x+2}$, line $3x + 2y = 0$ and the y–axis, about the x–axis. [4]

12 The curve **C** has equation $y = \dfrac{x}{1-x}$. State the equation of asymptotes of the curve.

Sketch the graph of **C**. [6]

By sketching the graph of $y = \ln(x + 2)$ on the same diagram as **C**, show that the equation $\dfrac{x}{1-x} = \ln(x + 2)$ has two real roots. Write down the two intervals in the form $(n, n + 1)$ containing the roots, where n is an integers. [4]

By taking $x = 1.5$ as the first approximation to the Newton-Raphson method, find the negative root correct to 3 significant figures. [5]

MATHEMATICS T (PAPER 2)

1. Find all the values of x, where $0^\circ < x < 360^\circ$, which satisfy the equation

$$2\sin\frac{x}{4} + \sin x + 2\sin\frac{7x}{4} = 0.$$ [5]

2. The positive variables of x and y are related by the differential equation $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^2$.

Find the general solution of this differential equation, expressing y in terms of x. [3]

Show that the member of the family of solution curves which passes through

the point (1, 1) is a straight line. [2]

3. By expressing $2\cos^2 x - 2\sin x\cos x - 1$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, sketch the graph of $y = 2\cos^2 x - 2\sin x\cos x - 1$, for $0 \le x \le 2\pi$. [7]

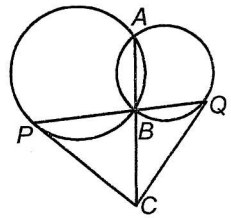
4. Refer to the origin O, the points A and B have position vectors $\underline{i} + 2\underline{j}$ and $2\underline{i} - 3\underline{j}$ respectively. The point P lies on AB such that $AP : PB = \lambda : 1 - \lambda$.

   Show that $\overrightarrow{OP} = (1 + \lambda)\underline{i} + (2 - 5\lambda)\underline{j}$. [3]

   (i) Find the value of λ for which OP is perpendicular to AB. [3]

   (ii) Find the value of λ for which $OA = AP$. [3]

5. In the figure, two circles intersect at A and B. AB is produced to C such that CP and CQ are tangents to the two circles respectively and PBQ is a straight line.



   (i) Express CP in terms of CB and CA, and hence show that CP = CQ,

   (ii) Show that the points A, P, C and Q are concyclic.

   (iii) If QA is produced to meet the larger circle at D, show that PB bisects ∠CPD. [11]

6. In a certain chemical reaction in which a compound X is formed from a compound Y. The masses of X and Y present at time t are x and y respectively. At any time t, the sum of the masses of X and Y is A, where A is a constant, and the rate at which x is increasing is proportional to the product of the two masses at that time.

   Show that $\frac{dx}{dt} = kx(A - x)$, where k is a constant.

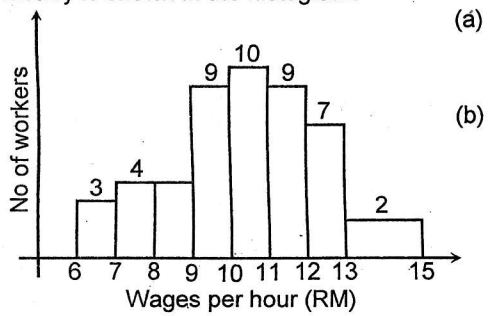
   If $x = \frac{A}{5}$ at $t = 0$, and $x = \frac{A}{2}$ at $t = \ln 2$, show that $k = \frac{2}{A}$.

   Hence, express x as a function of t. [9]

   (i) Find x and y in terms of A, at time $t = \ln 3$. [2]

   (ii) Find the value of t, correct to 3 significant figures when x is 99% of A. [2]

7. Wage per hour paid to 50 temporary workers in the production Department of a factory is shown in the histogram.



   (a) Calculate the mean wage per hour of the temporary workers in the Production Department of the factory. [2]

   (b) If the wage per hour of 25% of the workers is more than RM x, determine the value of x. [3]

8. A continuous random variable X has the probability density function,

$$f(x) = \begin{cases} k(4 - x^2) & , 0 \le x \le 2 \\ 0 & , \text{otherwise} \end{cases}$$

   , where k is a constant. Show that $k = \frac{3}{16}$. [2]

   Find the cumulative distribution function of X, and verify by calculation that the value of median of X lies between 0.69 and 0.70 [4]

9. An unbiased disc has a single dot marked on one side and two dots marked on the other side. The disc and an unbiased die are thrown and the random variable X is the sum of the number of dots shown on the disc and the score on the die.

   (a) Tabulate the probability distribution of X. [3]

   (b) Find the value of $P(X \ge 4 \mid X \le 7)$. [2]

   (c) Evaluate the mean and variance of X. [3]

10. Computer chips are difficult to manufacture, and need to be checked for faults before they can be used. A test procedure has two stages, and in each stage the probability of a faulty chip failing is 0.9. Any chip that fails the first stage is immediately rejected and does not go through the second stage of the test. Chips with no faults always pass both stages of the test. The probability of any randomly chosen chip being faulty is 0.4. Find the probability that

    (a) a randomly chosen chip is faulty and passes both stages of the test, [1]

    (b) a randomly chosen chip passes both stages of the test, [2]

    (c) a randomly chosen chip is faulty, given that it passes both stages of the test. [3]

11. (a) On a particular day in a year, the probability of a rainy day is 0.4.

    (i) Find the probability that in a particular week, it rains on the first two days and no rain on the rest of the week. [2]

    (ii) Find the probability that during a seven week holiday, there will be no rain. [2]

    (b) A shop sells two brands of computers. The average number of 'Matrix' sold per week is 1.8, and the corresponding figure for 'Tennou' is 2.7. Sales take place independently at random times. Find the probability that

    (i) exactly two 'Matrix' are sold in a given week, [1]

    (ii) exactly four 'Matrix' are sold in a given two week period, [2]

    (iii) the total number of computers sold in a given week is at least four. [4]

12. A maintenance company keeps records of the length of time it takes to repair faulty computers. The time that elapses between a customer reporting a computer out of action due to a fault and the fault being repaired is known as the 'repair time'. The company's records show that the repair time may be assumed to be normally distributed with mean 40 hours and standard deviation 5 hours, and the repair time on any particular occasion that a fault is reported may be assumed to be independent of all other repair times.

    (a) Find the probability that on a particular occasion the repair time will be less than 24 hours. [2]

    (b) The company wishes to state in its advertisements that 99% of all repair times will be less than a certain number of hours. Find the smallest integer that could truthfully be used in the advertisements. [3]

    (c) Find the probability that the sum of the repair times on two separate occasions, chosen at random, will be more than 100 hours. [3]

    (d) If a repair time is exceptionally long, the maintenance company pays compensation to the customer. The rate of compensation is RM 100 if the repair time is between 50 and 55 hours, and RM 250 if the repair time exceeds 55 hours. Find the expected amount of compensation per fault. [4]

1. $x^3-y^3+3xy-1=0$

$\therefore\ 3x^2-3y^2\cdot y'+3xy'+3y=0$

$\underline{(2,-1)}$: $12-3y'+6y'-3=0$

$\therefore\ y'=-3\ (=m)$ ✱

2. $P(-1)=-1+3-9+7=0$

$\therefore (x+1)$ is a factor of $P(x)$

$\therefore P(x)=(x+1)(x^2+2x+7)$

$=(x+1)[(x+1)^2+6]$

$\therefore (x+1)$ is the only linear factor of $P(x)$ ✱

3.(a) $w=\dfrac{20\sqrt{3}-12i}{\sqrt{3}-9i}\cdot\dfrac{\sqrt{3}+9i}{\sqrt{3}+9i}$

$=\dfrac{60+108+180\sqrt{3}i-12\sqrt{3}i}{3+81}=2+2i$ ✱

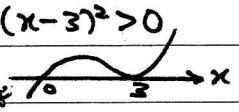
(b) $\underline{z=w^5}$: $z=2^5(1+i)^5$

$=32[1+5i-10-10i+5+i]$

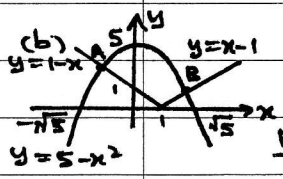
$=-128-128i$ ✱

4(a) $x^3>6x^2-9x \Rightarrow x(x-3)^2>0$

$\therefore \{x: 0<x<3 \text{ or } 3<x\}$ ✱



(b)



For A: $5-x^2=1-x$ $(x_A<0)$

$x^2-x-4=0$

$\therefore x_A=\frac{1}{2}(1-\sqrt{17})$

For B: $5-x^2=x-1$ $(x_B>0)$

$\therefore x^2+x-6=0$

$\therefore (x-2)(x+3)=0 \Rightarrow x_B=2$

$\therefore$ For $5-x^2\geq|1-x|$: $\{x: \frac{1}{2}(1-\sqrt{17})\leq x\leq 2\}$ ✱

5. $a=280$, $d=8$

For $T_n=1000$: $280+(n-1)(8)=1000$

$\therefore n=91$ ✱

Total $=\frac{91}{2}[280+1000]+13(1000)=71240$ ✱

6. (i) $\lim_{x\to 2^+} f(x)=3-\ln 2$

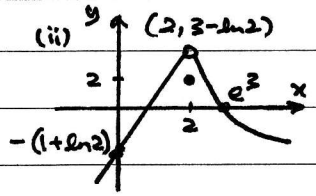
$\lim_{x\to 2^-} f(x)=2(2)-\ln 2-1$

$=3-\ln 2$

$\because \lim_{x\to 2^+} f(x)\neq 2$

$\therefore f$ is not continuous at $x=2$ ✱

(ii)



7.(a) $u=\sqrt{x}$: $u^2=x \Rightarrow 2u\frac{du}{dx}=1$

$\int\frac{\sin\sqrt{x}}{\sqrt{x}}dx=\int\frac{\sin u}{u}\cdot 2u\frac{du}{dx}=\int 2\sin u\,du$

$=-2\cos u+c=-2\cos\sqrt{x}+c$ ✱

(b) $\int_0^2 xe^{2x}dx=\left[\frac{x}{2}e^{2x}\right]_0^2-\int_0^2\frac{1}{2}e^{2x}dx$

$=e^4-0-\frac{1}{4}\left[e^{2x}\right]_0^2=e^4-\frac{1}{4}(e^4-1)=\frac{1}{4}[1+3e^{2x}]$ ✱

8(a) $A^2=\begin{bmatrix}-8&-6&-6\\2&0&2\\8&8&6\end{bmatrix}^2=\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$ ✱

(b) $A^{-1}=\frac{1}{4}A=\begin{bmatrix}-2&-3/2&-3/2\\1/2&0&1/2\\2&2&3/2\end{bmatrix}$ ✱

$\left.\begin{array}{l}-8x-6y-6z=13\\2x+2z=7\\8x+8y+6z=-25\end{array}\right\}$ $A\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}13\\7\\-25\end{pmatrix}$

$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{2}\begin{bmatrix}-4&-3&-3\\1&0&1\\4&4&3\end{bmatrix}\begin{pmatrix}13\\7\\-25\end{pmatrix}=\begin{pmatrix}1\\-6\\2.5\end{pmatrix}$

$\therefore x=1$, $y=-6$, $z=2.5$ ✱

9. $\sqrt{\frac{3+x}{3-x}}=(1+\frac{x}{3})^{\frac{1}{2}}(1-\frac{x}{3})^{-\frac{1}{2}}$

$=\left[1+\frac{1}{2}(\frac{x}{3})+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\frac{x}{3})^2+\dots\right]\left[1-\frac{1}{2}(-\frac{x}{3})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{x}{3})^2+\dots\right]$

$=(1+\frac{1}{6}x-\frac{1}{72}x^2+\dots)(1+\frac{1}{6}x+\frac{1}{54}x^2+\dots)$

$=1+\frac{1}{3}x+\frac{1}{18}x^2+\dots$ ✱

For expansion valid: $|\frac{x}{3}|<1 \Rightarrow |x|<3$

$\therefore \{x: -3<x<3\}$ ✱

$\underline{x=\frac{1}{5}}$: $\sqrt{\frac{16}{14}}\approx 1+\frac{1}{3}(\frac{1}{5})+\frac{1}{18}(\frac{1}{5})^2$

$\therefore \sqrt{14}\approx 4\left(\frac{450}{481}\right)=\frac{1800}{481}$ ✱

10. Let $D(a,b)$: $M_{BD}=M_{AC}$

$\left(\frac{a+4}{2},\frac{b+6}{2}\right)=\left(\frac{3}{2},\frac{6}{2}\right)\Rightarrow a=-1, b=0$

$\therefore D(-1,0)$ ✱

(i) $\underline{AC}$: $\frac{y-2}{x-1}=\frac{4-2}{2-1}\Rightarrow y=2x$

Let $E(e,2e)$: $m_{BE}=-\frac{1}{2}=\frac{2e-6}{e-4}$

$\therefore e=\frac{16}{5}\Rightarrow E\left(\frac{16}{5},\frac{32}{5}\right)$ ✱

(ii) Area $=\frac{1}{2}\begin{vmatrix}1&4&2&1\\2&6&4&2\end{vmatrix}\times 2$

$=|(6+16+4)-(8+12+4)|=2$ ✱

(iii) $\angle AEB=90^\circ$: AB (diameter)

$\left(\frac{y-2}{x-1}\right)\left(\frac{y-6}{x-4}\right)=-1\Rightarrow x^2+y^2-5x-8y+16=0$

(circle eq.) ✱

11(a) Area $\simeq\frac{1}{2}(\frac{1}{4})\left[0+\frac{1}{2}+2\left(\frac{0.25}{1.0625}+\frac{0.5}{1.25}+\frac{0.75}{1.5625}\right)\right]=0.34132$ (5dp) ✱

$0.34132\simeq\int_0^1\frac{x}{1+x^2}dx=\frac{1}{2}\left(\ln(1+x^2)\right]_0^1$

$\therefore 0.34132\approx\frac{1}{2}\ln 2\Rightarrow \ln 2=0.683$ (3dp) ✱

(b)(i) $T(-\frac{2}{3},1)$: $1=e^{3(-\frac{2}{3})+1}=e^0=1$ [consistent]

$3(-\frac{2}{3})+2(1)=0$ [consistent]

$\therefore T$ is the intersection point. ✱

(ii) $V_x=\pi\int_{-\frac{2}{3}}^0\left[e^{3x+2}\right]^2dx-\frac{\pi}{3}(\frac{2}{3})(1)^2$

$=\frac{\pi}{6}\left[e^{2(3x+2)}\right]_{-2/3}^0-\frac{2}{9}\pi=\frac{\pi}{18}[3e^4-7]$ ✱

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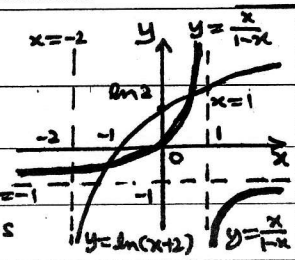
12. $y = \frac{x}{1-x} = \frac{1}{1-x} - 1$



asymptotes: $x=1, y=-1$

$y=\frac{x}{1-x}$ and $y=\ln(x+2)$ has 2 intersection points

$\therefore \frac{x}{1-x} = \ln(x+2)$ has 2 real roots (α and β)

$\Rightarrow -2<\alpha<-1$ and $0<\beta<1$ ✱

Let $f(x) = \frac{x}{1-x} - \ln(x+2)$:

$f'(x) = \frac{1}{(1-x)^2} - \frac{1}{x+2}$

By NRM: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1=-1.5$: $x_2 = -1.5 - \frac{0.09315}{-1.84}$

$= -1.4494 \simeq -1.45$

$x_3 = -1.4494 - \frac{0.00501}{-1.6495} = -1.4464 \simeq -1.45$

$\therefore \alpha = -1.45$ (3s.f) ✱

1. $2\sin\frac{x}{4} + \sin x + 2\sin\frac{7x}{4} = 0$

$\therefore 2\left[2\sin x\cos\frac{3x}{4}\right] + \sin x = 0$

$\therefore \sin x = 0$ or $\cos\frac{3x}{4} = -\frac{1}{4}$

$\therefore x = 180°$ or $\frac{3x}{4} = 104.48, 255.52$

$\therefore x = 180°, 139.3°, 340.7°$ ✱

2. $\frac{dy}{dx} = \left(\frac{y}{x}\right)^2 \Rightarrow \int \frac{dy}{y^2} = \int \frac{dx}{x^2}$

$\therefore -\frac{1}{y} = -\frac{1}{x} + c \Rightarrow y = \frac{x}{1-cx}$ ✱

(1,1): $c=0 \Rightarrow y=x$ (a straight line) ✱

3. $f(x) = 2\cos^2 x - 2\sin x\cos x - 1$

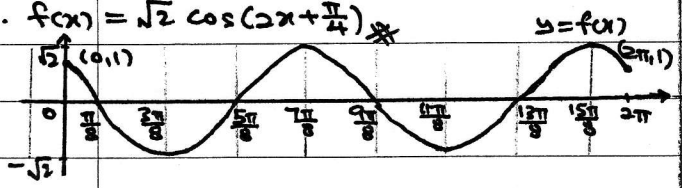
$= \cos 2x - \sin 2x = R\cos(2x+\alpha)$

$= R\cos 2x\cos\alpha - R\sin 2x\sin\alpha$

$\therefore R\cos\alpha = 1, R\sin\alpha = 1$

$\therefore \alpha = \tan^{-1}(1) = \frac{\pi}{4}$ $\therefore R\left(\frac{1}{\sqrt{2}}\right) = 1 \Rightarrow R = \sqrt{2}$

$\therefore f(x) = \sqrt{2}\cos\left(2x + \frac{\pi}{4}\right)$ ✱



4. $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\overrightarrow{AB}$

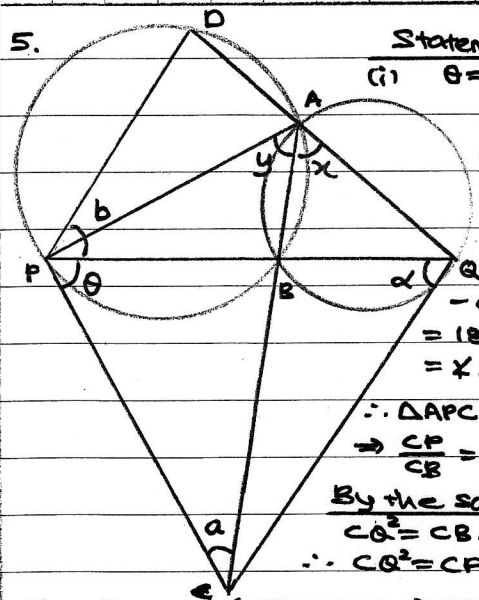
$= \begin{pmatrix}1\\2\end{pmatrix} + \lambda\left[\begin{pmatrix}2\\-3\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}\right] = \begin{pmatrix}1+\lambda\\2-5\lambda\end{pmatrix}$ ✱

(i) $OP \perp AB$: $\begin{pmatrix}1+\lambda\\2-5\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\-5\end{pmatrix} = 0$

$\therefore 1+\lambda-10+25\lambda = 0 \Rightarrow \lambda = \frac{9}{26}$ ✱

(ii) $|\overrightarrow{OA}| = |\overrightarrow{AP}|$: $\sqrt{5} = \lambda[\sqrt{26}] \Rightarrow \lambda = \sqrt{\frac{5}{26}}$ ✱

5.



Statements	Reasons
(i) $\theta = y$	Angles of Alternate segment.
$\angle PCB = \angle ACP = a$	common angle.
$\angle PBC = 180° - a - \theta = 180° - a - y = \angle APC$	sum of angles in Δ / sum of angles in Δ

$\therefore \Delta APC \sim \Delta PBC$

$\Rightarrow \frac{CP}{CB} = \frac{CA}{CP} \Rightarrow CP^2 = CB.CA$

By the same method:

$CQ^2 = CB.CA$

$\therefore CQ^2 = CP^2 \Rightarrow CQ = CP$ ✱

(ii) $\theta = \alpha$ ($CP=CQ \Rightarrow \Delta CPQ$ is isosceles)

$\therefore \theta = x$

$\therefore$ A pair of equal angle in same segment.

$\Rightarrow$ APCQ are concyclic. ✱

(iii) $b = x$ (exterior angle of cyclic PBAD)

$\therefore b = \theta \Rightarrow$ PB bisects $\angle CPD$ ✱

6. $x+y=A$: $\frac{dx}{dt} \propto xy \Rightarrow \frac{dx}{dt} = kx(A-x)$ ✱

$\therefore \int_{A/5}^{x} \frac{1}{x(A-x)}dx = \int_0^t k\,dt$

$\therefore \int_{A/5}^{x}\left[\frac{1}{A-x} + \frac{1}{x}\right]dx = \int_0^t Ak\,dt$

$\therefore \left[\ln\left(\frac{x}{A-x}\right)\right]_{A/5}^{x} = Ak[t]_0^t$

$\therefore \ln\left(\frac{4x}{A-x}\right) = Akt$

$t=\ln 2$: $\ln\left(\frac{2A}{A/2}\right) = Ak\ln 2 = 2\ln 2$

$\therefore k = \frac{2}{A}$ ✱

$\therefore \frac{4x}{A-x} = e^{2t} \Rightarrow \frac{A}{x} - 1 = 4e^{-2t}$

$\therefore x = \frac{A}{1+4e^{-2t}}$ ✱

(i) $t=\ln 3$: $x = A/1+4\left(\frac{1}{9}\right) = \frac{9}{13}A$ $\therefore y = \frac{4}{13}A$ ✱

(ii) $x = 0.99A$: $e^{2t} = \frac{4(0.99)}{1-0.99} \Rightarrow t = \frac{1}{2}\ln 396$

$\therefore t = 2.99$ (3s.f) ✱

7. (i) $\bar{x} = \frac{3(6.5)+4(7.5+8.5)+9(9.5+11.5)+10(10.5)+7(12.5)+2(14)(2)}{50}$

$= RM10.24$ ✱

(ii) $x = 11 + \left(\frac{37.5-30}{9}\right)(1) = 11.83$ ✱

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8. $\int_0^2 k(4-x^2)\,dx = 1 = k\left[4x - \frac{x^3}{3}\right]_0^2$

$\therefore k\left(8 - \frac{8}{3}\right) = 1 \Rightarrow k = \frac{3}{16}$ *

$P(X \le x) = \int_0^x \frac{3}{16}(4-x^2)\,dx$

$= \frac{3}{16}\left[4x - \frac{x^3}{3}\right]_0^x = \frac{3}{16}\left(4x - \frac{x^3}{3}\right)$

$\therefore F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{16}\left(4x - \frac{x^3}{3}\right) & 0 \le x \le 2 \\ 1 & 2 < x \end{cases}$ *

$F(0.69) = 0.497 \ (<0.5)$, $F(0.70) = 0.504 \ (>0.5)$

$\therefore 0.69 < m < 0.70$ *

9. (i)

X	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8

x	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

(ii) $P(X \ge 4 \mid X \le 7) = \frac{P(4 \le X \le 7)}{P(X \le 7)}$

$= 4\left(\frac{2}{12}\right) \Big/ 1 - \frac{1}{12} = \frac{8}{11}$ *

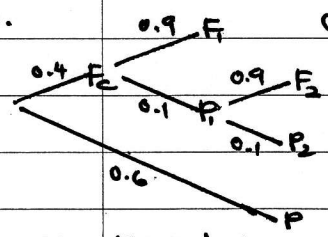
(iii) Distribution is symmetrical : $E(X) = 5$ *

$E(X^2) = \frac{1}{12}\left[2^2(1) + (3^2+4^2+5^2+6^2+7^2)(2) + 8^2(1)\right]$

$= 28\frac{1}{6}$

$Var(X) = 28\frac{1}{6} - 5^2 = \frac{19}{6}$ *

10.



(i) Prob. $= 0.4(0.1)^2$

$= 0.004$ *

(ii) Prob $= 0.004 + 0.6$

$= 0.604$ *

(iii) $P(F_c \mid P) = \frac{0.004}{0.604} = \frac{1}{151}$ *

11. (a) (i) Prob. $= (0.4)^2(0.6)^5 = 0.01244$ *

(ii) Prob $= (0.6)^7 = 0.02799$ *

(b) (i) Prob. $= e^{-1.8}\left(\frac{1.8^2}{2!}\right) = 0.2678$ *

(ii) Prob. $= e^{-3.6}\left(\frac{3.6^4}{4!}\right) = 0.1912$ *

(iii) Prob $= 1 - e^{-4.5}\left[1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!}\right]$

$= 0.6577$ *

12. X = repair time, $X \sim N(40, 5^2)$

(i) $P(X < 24) = P\left(Z < \frac{24-40}{5}\right) = 0.00669$ *

(ii) $P(X < k) = 0.99$: $\frac{k-40}{5} = z_{0.01} = 2.326$

$\therefore k = 51.63 \simeq 52$ *

(iii) $P(X_1 + X_2 > 100) = P\left(Z > \frac{100-80}{5\sqrt{2}}\right)$

$= P(Z > 2.828) = 0.00234$ *

(iv) E(compensation)

$= 100 \cdot P(50 < X < 55) + 250 \cdot P(X > 55)$

$= 100 \cdot P(2 < Z < 3) + 250\, P(Z > 3)$

$= 100\left[0.0228 - 0.0014\right] + 250\,(0.0014)$

$= \text{RM}\,2.49$ *

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