TAR COLLEGE (2005)
MATHEMATICS T (PAPER 1)

1. Prove that $x^2 + y^2 > 2xy$ for all real values of x and y, and $x \neq y$. [2]

   If $x^2 + y^2 = 1$ and $w^2 + z^2 = 1$, show that $xw + yz < 1$. [2]

2. By using the laws of set algebra, prove that, for any sets A and B,

   $[A \cap (A' \cup B)] \cup [B \cap (A' \cup B')] = B$ [4]

3. If $\frac{1}{b-2a}, \frac{1}{b}, \frac{1}{b-2c}$ are in an arithmetic progression, show that $a(b-c)$, ac, $c(b-a)$ are also in an arithmetic progression. [5]

4. Given that the matrix $\begin{pmatrix} a-2 & 4 & 3 \\ 1 & a+1 & -2 \\ 0 & 0 & a-4 \end{pmatrix}$ is singular. Find the possible values of a. [4]

5. The function f is defined by $f : x \longrightarrow \begin{cases} 2|x-4| & , x < 2 \\ 4x - x2 & , x \geq 2 \end{cases}$.

   (i) Without using graph, determine whether f is continuous at $x = 2$. [4]

   (ii) Sketch the graph of f. [3]

6. Find the values of A, B and C if $\frac{x^2+6x+7}{(x+2)(x+3)} \equiv A + \frac{B}{x+2} + \frac{C}{x+3}$. [4]

   Hence, show that $\int_0^1 \frac{x^2+6x+7}{(x+2)(x+3)}\,dx = 1 + \ln\frac{32}{27}$. [4]

7. A rhombus has vertices $A(1, 4)$, $B(-3, -3)$, $C(a, b)$ and $D(9, 5)$. Find the values of a and b. [4]

   $E(1, -\frac{5}{2})$ is a point on BC and $F(7, \frac{3}{2})$ is a point on CD. Find the shortest distance A from EF, and the area of the triangle AEF. [6]

8. If $\frac{a}{b}$ and $\frac{a}{c}$ are small, show that by ignoring the powers higher than square and products of these quantities, $\sqrt{\frac{b+a}{c+a}} = \left[1 + \frac{a}{2}\left(\frac{1}{b} - \frac{1}{c}\right)\right]\sqrt{\frac{b}{c}}$. [4]

   By using this result, find an approximation for $\sqrt{\frac{101}{104}}$ in the form $\frac{m}{n}$, where m and n are integers. [4]

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9. Given that $f(x) = x^4 - 4x^3 + ax^2 + bx - 16$, where a and b are constants. If $(x + 2)$ is a factor of $f(x)$ and $(x - 3)$ is a factor of $f'(x)$, find the values of a and b. [5]

   With these values of a and b, factorise $f(x)$ completely. [5]

   Hence, solve the equation of $f(x) = 0$. [4]

10. By sketching the graphs of $y = x^3$ and $y = 3x - 1$ on the same coordinate axes, show that the equation $x^3 - 3x + 1 = 0$ has three real roots.
    Show that one of the roots lies between 0 and 1. [7]

    By using the Newton-Raphson method, find this root, correct to two decimal places. [6]

11. Find the coordinates of the points of intersection between $y^2 = x$ and $x + y = 2$. [3]

    (i) Find the area bounded by the two curves. [5]

    (ii) Find the volume of the solid generated when this area is rotated through four right angles about the y–axis. [5]

12. Evaluate $\int_2^4 \frac{\ln x}{x}\,dx$, giving your answers correct to three decimal places, by using

    (i) the substitution $y = \ln x$. [5]

    (ii) the trapezium rule with five ordinates. [5]

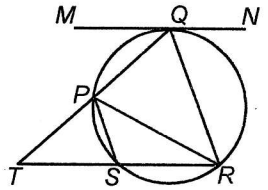
MATHEMATICS T (PAPER 2)

1. At an instant, a ship P is 2 km to the north of ship Q. Ship P sails at a speed of 3 ms^{-1} to east, and ship Q sails at 4 ms^{-1} in the direction $N\,60^\circ\,E$. Find

   (a) the magnitude and direction of the velocity of Q relative to P. [4]

   (b) the time taken, to the nearest minute, for Q to be due east of P. [3]

2. In the figure, MQN is a tangent to the circle at Q. The chord RS is parallel to MN. The chords PQ and RS are produced to meet at T. Show that



   (a) $\angle SPT = \angle QRS$ [2]

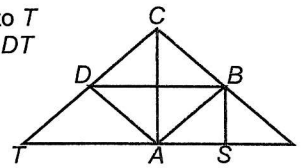
   (b) $\angle QRS = \angle QPR$ [2]

   (c) the triangles PQR and PTS are similar. [3]

3. Solve the differential equation, $\frac{dy}{dx} = \frac{y^2 + xy}{x^2}$ given that $y = 2$ when $x = 1$. [5]

4. In the figure, $ABCD$ is a rhombus and CD is produced to T such that $CD = DT$. Show that the triangles BCD and ADT are congruent, and the diagonal BD is parallel to TA.

If the line parallel to AC through B is produced to meets TA produced at S, show that $\angle ASB = 90^\circ$. [6]



5. (a) Using the formula of $\cos 3\theta$ and $\sin 3\theta$, show that for all values of θ,

$$4(\cos 3\theta \sin^3 \theta + \sin 3\theta \cos^3 \theta) = 3 \sin 4\theta.$$ [5]

(b) Express $\cos 2x - \sin 2x$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence, find all the values of x in the range $0 \le x \le 2\pi$, such that

$$\cos 2x - \sin 2x = 1.$$ [6]

6. In a village of N villagers, the number of villagers heard of a rumour is x at time t, and the rate of spreading the rumour is given by $\frac{dx}{dt} = \frac{1}{250}x(N - x)$.

If t is the number of days, and at $t = 0$, there were two villagers in 1000 villages started spreading the rumour, find

(a) x in terms of t, [7]

(b) the time taken for 50% of the villagers heard the rumour. [4]

7. A bag contains 5 golf balls and 3 other balls, whereas a box contains 4 golf balls and 2 other balls. A die is thrown and if the score is less than 5, a ball is drawn from the bag, otherwise a ball is drawn from the box. Find the probability that

(a) the ball drawn is a golf ball, [3]

(b) if the ball drawn is a golf ball, it came from the box. [3]

8. Between 10 am and 11 am, Directory Enquiries receives calls at rate of 3 per minute. Assuming that the calls received at random point in time, determine the probability that

(a) 5 calls received in a randomly chosen minute, [2]

(b) not more 3 calls received in a randomly chosen minute, [3]

(c) 8 calls received in a randomly chosen three minutes period. [2]

9. The table shows the diameter, to the nearest millimeter, of a batch of ball bearings produced by a machine.

Diameter (mm)	Frequency
10 – 14	7
15 – 24	22
25 – 29	8
30 – 36	9
37 – 42	5
43 – 54	5
55 – 64	3

(a) Draw a histogram to represent this information. [6]

(b) Find the mean and standard deviation. [5]

(c) Estimate the median and semi-interquartile range. [3]

(d) Explain why median is better than the mean as a measure of central tendency for the data above. [1]

10. Given that the random variable X has a normal distribution. If $P(X > 9) = 0.9032$ and $P(X < 11) = 0.7257$, determine the values of $P(X > 10)$. [8]

11. A discrete random variable X has the following probability distribution.

$$P(X = x) = \begin{cases} k\left|\frac{1}{3} - x\right| & \text{, for } x = 0,1,2 \\ \frac{1}{5x} & \text{, for } x = 3 \\ 0 & \text{, otherwise} \end{cases}$$

(a) Determine the value of k. [2]

(b) Find $E(X)$ and $Var(X)$. [3]

(c) Find $Var(3X - 5)$. [2]

12. A continuous random variable X has the probability density function,

$$f(x) = \begin{cases} \frac{1}{3} & \text{, } -1 \le x \le 0 \\ \frac{1}{3} + a & \text{, } 0 < x \le 1 \\ 0 & \text{, otherwise} \end{cases} \text{, where } a \text{ is a constant}$$

(a) Find the value of a. [2]

(b) Find the cumulative distribution function of X and sketch its graph. [4]

(c) Calculate the mean and standard deviation of X. [4]

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1. $(x-y)^2>0$, $(\forall x,y\in\mathbb{R},\ x\neq y)$

$\therefore\ x^2-2xy+y^2>0 \Rightarrow x^2+y^2>2xy$ *

$x^2+y^2=1,\ w^2+z^2=1\ :\ x^2+y^2+w^2+z^2=2$

$x^2+y^2+w^2+z^2>2xy+2wz$

$\therefore\ xy+wz<1$ *

2. $[A\cap(A'\cup B)]\cup[B\cap(A'\cup B')]$

$=[(A\cap A')\cup(A\cap B)]\cup[(B\cap A')\cup(B\cap B')]$

$=[\emptyset\cup(A\cap B)]\cup[(A'\cap B)\cup\emptyset]$

$=(A\cap B)\cup(A'\cap B)=(A\cup A')\cap B$

$=\mathcal{E}\cap B=B$ *

3. AP: $\frac{1}{b-2a}+\frac{1}{b-2c}=2\left(\frac{1}{b}\right)$

$\therefore\ \frac{2b-2a-2c}{b^2-2ab-2cb+4ac}=\frac{2}{b}$

$\therefore\ b^2-ab-bc=b^2-2ab-2bc+4ac$

$\therefore\ ab-2ac+bc=2ac$

$\therefore\ a(b-c)+c(b-a)=2ac$

$\therefore\ a(b-c),\ ac,\ c(b-a)$ also in AP *

4. $\begin{bmatrix} a-2 & 4 & 3 \\ 1 & a+1 & -2 \\ 0 & 0 & a-4 \end{bmatrix}$ singular $\Rightarrow \det=0$

$\therefore (a-4)[(a-2)(a+1)-4]=0$

$\therefore\ a=4$ or $a^2-a-6=0=(a+2)(a-3)$

$\therefore\ a=-2,3$ or 4 *

5. (i) $f(2)=4(2)-2^2=4$

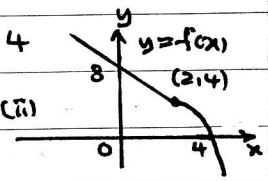
$\lim_{x\to2^-}f(x)=2|2-4|=4$

$\lim_{x\to2^+}f(x)=4(2)-2^2=4$

$\therefore\ \lim_{x\to2}f(x)=f(2)$

$\Rightarrow f$ is continuous at $x=2$. *

(ii)



6. $\frac{x^2+6x+7}{(x+2)(x+3)}\equiv A+\frac{B}{x+2}+\frac{C}{x+3}=f(x)$

$\equiv\frac{A(x+2)(x+3)+B(x+3)+C(x+2)}{(x+2)(x+3)}$

$\therefore\ x^2+6x+7\equiv A(x+2)(x+3)+B(x+3)+C(x+2)$

$[x^2]:\ 1=A$ * $\quad x=-2:\ 4-12+7=B\Rightarrow B=-1$ *

$[x^0]:\ 7=6A+3B+2C\Rightarrow C=2$ *

$\therefore\ \int_0^1 f(x)\,dx=\int_0^1\left[1-\frac{1}{x+2}+\frac{2}{x+3}\right]dx$

$=\left[x-\ln(x+2)+2\ln(x+3)\right]_0^1$

$=(1-0)-\ln\left(\frac{3}{2}\right)+2\ln\left(\frac{4}{3}\right)$

$=1+\ln\left[\left(\frac{4}{3}\right)^2\left(\frac{2}{3}\right)\right]=1+\ln\left(\frac{32}{27}\right)$ *

7. $M_{AC}=M_{BD}$: $\left(\frac{a+1}{2},\frac{b+4}{2}\right)=\left(\frac{a-3}{2},\frac{5-3}{2}\right)$

$\therefore\ a+1=6$ and $b+4=2\Rightarrow a=5,\ b=-2$ *

EF: $\frac{y+\frac{5}{2}}{x-1}=\frac{4}{6}\Rightarrow 4x-6y-19=0$

$\therefore$ distance A from EF $=\frac{|4(1)-6(4)-19|}{\sqrt{4^2+6^2}}$

$=\frac{39}{2\sqrt{13}}=\frac{3}{2}\sqrt{13}$ *

Area $\Delta AEF=\frac{1}{2}\left[\frac{3}{2}\sqrt{13}\right]\left[\sqrt{4^2+6^2}\right]=\frac{39}{2}$ units^2 *

8. $\sqrt{\frac{b+a}{c+a}}=\sqrt{b}\left(1+\frac{a}{b}\right)^{\frac{1}{2}}\cdot\frac{1}{\sqrt{c}}\left(1+\frac{a}{c}\right)^{-\frac{1}{2}}$

$=\sqrt{\frac{b}{c}}\left[1+\frac{1}{2}\left(\frac{a}{b}\right)+\dots\right]\left[1-\frac{1}{2}\left(\frac{a}{c}\right)+\dots\right]$

$=\left[1+\frac{a}{2}\left(\frac{1}{b}-\frac{1}{c}\right)\right]\sqrt{\frac{b}{c}}$ *

Let $b=100,\ c=25,\ a=1$:

$\sqrt{\frac{101}{26}}=\left[1+\frac{1}{2}\left(\frac{1}{100}-\frac{1}{25}\right)\right]\frac{10}{5}=\frac{197}{100}=\times\left(\frac{1}{2}\right)$

$\therefore\ \sqrt{\frac{101}{104}}=\frac{197}{200}$ *

9. $f(x)=x^4-4x^3+ax^2+bx-16$

$\therefore\ f'(x)=4x^3-12x^2+2ax+b$

$f(-2)=0:\ 16+32+4a-2b-16=0\Rightarrow 2a-b=-16$

$f'(3)=0:\ 108-108+6a+b=0\Rightarrow 6a+b=0$

$\therefore\ 8a=-16\Rightarrow a=-2\quad\therefore\ b=12$ *

(i) $f(x)=x^4-4x^3-2x^2+12x-16$

$f(4)=256-256-32+48-16=0$

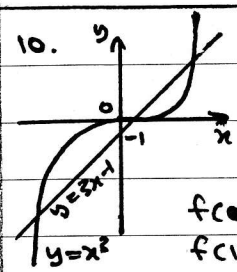
$\therefore\ (x-4)$ is a factor of $f(x)$

$\therefore\ f(x)=(x+2)(x-4)(x^2-2x+2)$ *

(ii) $f(x)=0$: $x=-2,4$ or $\frac{2\pm\sqrt{4-8}}{2}$

$\therefore\ x=-2,4$ or $1\pm i$ *

10.



$y=x^3$ intersect $y=3x-1$ at 3 points $\Rightarrow x^3-3x+1=0$ has 3 real roots *

Let $f(x)=x^3-3x+1$

$f(0)\cdot f(1)=(1)(-1)\ (<0)$

$f(1)\cdot f(2)=(-1)(3)\ (<0)$

$\therefore$ exists 1 real root $\in(0,1)$ and $\in(1,2)$ each

$\therefore$ one of the root $\in(0,1)$ *

$f'(x)=3x^2-3$, By NRM: $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

$x_0=0.3:\ x_1=0.3-\frac{0.127}{(-2.73)}=0.347\approx0.35$

$x_2=0.347-\frac{0.00078}{(-2.639)}=0.347\approx0.35$

$\therefore$ root $=0.35$ (2 dp) *

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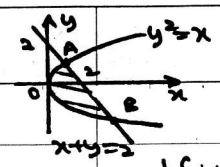
11.



For A, B: $y^2 = 2-y$

$\therefore y^2+y-2=0=(y-1)(y+2)$

$\therefore y=1$ or -2

$\therefore A(1,1)$, $B(4,-2)$ ✱

(i) Area $= \frac{1}{2}[1+4](3) - \int_{-2}^{1} y^2\,dy$

$= \frac{15}{2} - \left[\frac{1}{3}y^3\right]_{-2}^{1} = \frac{15}{2} - \frac{1}{3}[1+8] = \frac{9}{2}$ ✱

(ii) $V_y = \pi\int_{-2}^{1}\left[(2-y)^2 - y^4\right]dy$

$= \pi\left[\frac{(2-y)^3}{-3} - \frac{1}{5}y^5\right]_{-2}^{1}$

$= \pi\left[-\frac{1}{3}(1-64) - \frac{1}{5}(1+32)\right] = \frac{72}{5}\pi$ ✱

12. (i) $y=\ln x$: $\frac{dy}{dx} = \frac{1}{x}$

$\int_2^4 \frac{\ln x}{x}\,dx = \int_{\ln 2}^{2\ln 2} y\,dy = \left[\frac{1}{2}y^2\right]_{\ln 2}^{2\ln 2}$

$= \frac{1}{2}\left[(2\ln 2)^2 - (\ln 2)^2\right] = 0.721$ ✱

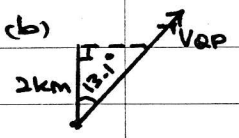
(ii) $\int_2^4 \frac{\ln x}{x}\,dx = \frac{1}{2}(0.5)\left(\frac{\ln 2}{2} + \frac{\ln 4}{4} + 2\left[\frac{\ln 2.5}{2.5} + \frac{\ln 3}{3} + \frac{\ln 3.5}{3.5}\right]\right) = 0.719$ ✱

1. (a) $\underline{V_{QP}} = 4\begin{pmatrix}\sin 60^\circ\\ \cos 60^\circ\end{pmatrix} - 3\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0.464\\2\end{pmatrix}$

$= \sqrt{0.464^2+2^2}$, to N $\tan^{-1}\left[\frac{0.464}{2}\right]$ E

$= 2.053\,ms^{-1}$, to N 13.1° E ✱

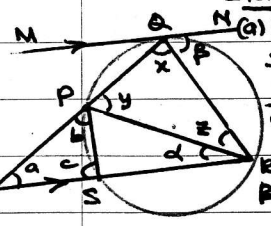
(b)



time taken $= \frac{2000}{\cos 13.1^\circ} \Big/ 2.053$

$= 1000s = 16min\ 40sec.$ ✱

2.



Statement	Reason
(a) $b=\alpha+z$	Exterior angle of cyclic PQRS
$\therefore \angle SPT = \angle QRS$ ✱	
(b) $y=\beta$	Angles of alternate segment
$\beta=\alpha+z$	Alternate angles (MN // TR)
$\therefore y=\alpha+z \Rightarrow \angle QPR = \angle QRS$ ✱	

(c) $b=y$	by (a), (b)
$c=x$	Exterior angle of cyclic PQRS
$a=180^\circ-c-b$	sum of angles in ΔPTS
$=180^\circ-x-y$	
$=z$	sum of angles in ΔPQR
$\therefore \Delta PRQ \sim \Delta PTS$ ✱	

3. Let $u=xy$: $\frac{y^2+xy}{x^2} = \frac{dy}{dx}$

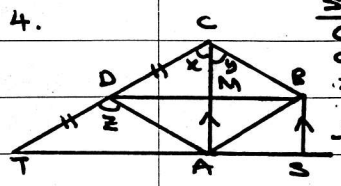
$\therefore u^2+u = u + x\frac{du}{dx} \Rightarrow \int_2^u \frac{1}{u^2}\,du = \int_1^x \frac{1}{x}\,dx$

$\therefore \left(-\frac{1}{u}\right]_2^u = [\ln x]_1^x \Rightarrow \frac{1}{2} - \frac{1}{u} = \ln x$

$\therefore \frac{xy-2}{2xy} = \ln x \Rightarrow xy-2 = 2xy\ln x$

$\therefore y = \frac{2}{x(1-2\ln x)}$ ✱

4.



Statement	Reason
$CD=DT$	given
$CB=DA$	Rhombus ABCD
$x+y=z$	Corresponding angles, BC // AD
$\therefore \Delta BCD \equiv \Delta ADT$ ✱	By SAS

$\angle BMA = 90^\circ$	ABCD – rhombus
$\angle CDB = \angle DTA$	$\Delta CDB \equiv \Delta DTA$
$\Rightarrow DB // TS$	Corresponding angles equal
$\therefore \angle ASB = \angle BMA = 90^\circ$ ✱	ASBM is a parallelogram

5. (a) $4[\cos 3\theta\sin^3\theta + \sin 3\theta\cos^3\theta]$

$= \cos 3\theta[3\sin\theta - \sin 3\theta] + \sin 3\theta[3\cos\theta + \cos 3\theta]$

$= 3[\sin 3\theta\cos\theta + \cos 3\theta\sin\theta]$

$= 3\sin(3\theta+\theta) = 3\sin 4\theta$ ✱

(b) $f(x) = \cos 2x - \sin 2x = R\cos(2x+\alpha)$

$= R\cos 2x\cos\alpha - R\sin 2x\sin\alpha$

$R\cos\alpha = 1$, $R\sin\alpha = 1$ | $\therefore R\left(\frac{1}{\sqrt{2}}\right) = 1$

$\therefore \tan\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$ | $\therefore R = \sqrt{2}$

$\therefore f(x) = \sqrt{2}\cos\left(2x - \frac{\pi}{4}\right)$ ✱

For $f(x)=1$: $\cos\left(2x-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\therefore 2x - \frac{\pi}{4} = -\frac{\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$

$\therefore x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$ ✱

6. $\frac{dx}{dt} = px(N-x)$, $p=\frac{1}{250}$, $N=1000$

(a) $\int_2^x \frac{dx}{x(N-x)} = \int_0^t p\,dt$

$\therefore \int_2^x \left[\frac{1}{x} + \frac{1}{N-x}\right]dx = pN[t]_0^t$

$\therefore \left[\ln\left(\frac{x}{N-x}\right)\right]_2^x = pNt = \ln\frac{x}{N-x} - \ln\frac{2}{998}$

$\therefore 4t = \ln\left[\frac{499x}{1000-x}\right] \Rightarrow \frac{499x}{1000-x} = e^{4t}$

$\therefore x = \frac{1000e^{4t}}{499+e^{4t}} = \frac{1000}{499e^{-4t}+1}$ ✱

(b) $\underline{x=500}$: $t = \frac{1}{4}\ln\left[\frac{499(500)}{500}\right] = \frac{1}{4}\ln 499$

$= 1.55$ days ✱

7. (a) $P(G) = \frac{4}{6}\left(\frac{5}{8}\right) + \frac{2}{6}\left(\frac{4}{6}\right) = \frac{23}{36}$ ✱

(b) $P(\text{Box}\,|\,G) = \frac{2}{6}\left(\frac{4}{6}\right) \Big/ \frac{23}{36} = \frac{8}{23}$ ✱

8. X = no. of calls in a minute, $X \sim Po(3)$

(a) $P(X=5) = e^{-3}\left[\frac{3^5}{5!}\right] = 0.1008$ ✱

(b) $P(X\le 3) = e^{-3}\left[1+3+\frac{3^2}{2!}+\frac{3^3}{3!}\right] = 0.6472$ ✱

(c) $T = X_1+X_2+X_3$: $T \sim Po(9)$

$P(T=8) = e^{-9}\left[9^8/8!\right] = 0.1318$ ✱

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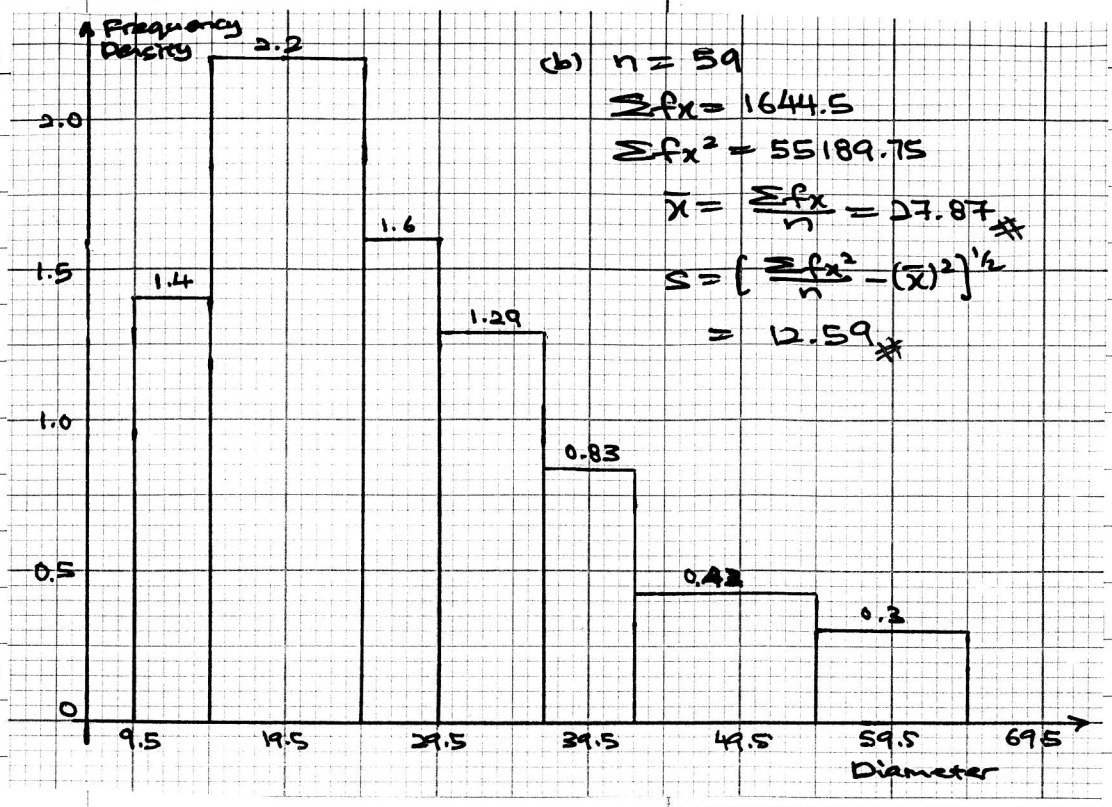
9.(a)



(b) $n = 59$

$\Sigma fx = 1644.5$

$\Sigma fx^2 = 55189.75$

$\bar{x} = \frac{\Sigma fx}{n} = 27.87$ ✱

$s = \left[\frac{\Sigma fx^2}{n} - (\bar{x})^2\right]^{1/2}$

$= 12.59$ ✱

(c) $m = x_{29.5} = 24.5 + \left(\frac{29.5-29}{8}\right)(5)$

$= 24.81$ ✱

$SIQR = (Q_3 - Q_1)\left(\frac{1}{2}\right)$

$= \frac{1}{2}\left[\left(29.5 + \frac{44.25-37}{9}(7)\right) - \left(14.5 + \frac{14.75-7}{22}(10)\right)\right] = 8.56$ ✱

(d) median is better because the distribution is right-skewed. ✱

10. $X \sim N(\mu, \sigma^2)$

$P(X>9) = 0.9032$: $\frac{9-\mu}{\sigma} = -z_{0.0968} = -1.3$

$\therefore\ 9 - \mu = -1.3\sigma$ — ①

$P(X<11) = 0.7257$: $\frac{11-\mu}{\sigma} = z_{0.2743} = 0.6$

$\therefore\ 11 - \mu = 0.6\sigma$ — ②

② − ① : $2 = 1.9\sigma \Rightarrow \sigma = 1.053$

$\therefore\ \mu = 11 - 0.6\left(\frac{2}{1.9}\right) = 10.368$

$P(X>10) = P\left(Z > \frac{10-10.368}{1.053}\right) = 0.6366$ ✱

11. (a) $\Sigma P(X=x) = 1$:

$k\left[\frac{1}{3} + \frac{2}{3} + \frac{5}{3}\right] + \frac{1}{15} = 1 \Rightarrow k = \frac{7}{20}$ ✱

(b) $E(X) = \frac{7}{20}\left[\frac{1}{3}(0) + \frac{2}{3}(1) + \frac{5}{3}(2)\right] + \frac{1}{15}(3) = 1.6$ ✱

$E(X^2) = \frac{7}{20}\left[\frac{1}{3}(0)^2 + \frac{2}{3}(1)^2 + \frac{5}{3}(2)^2\right] + \frac{1}{15}(3)^2 = \frac{19}{6}$

$\therefore\ Var(X) = \frac{19}{6} - 1.6^2 = \frac{91}{150}$ ✱

(c) $Var(3X-5) = 3^2\left(\frac{91}{150}\right) = 5.46$ ✱

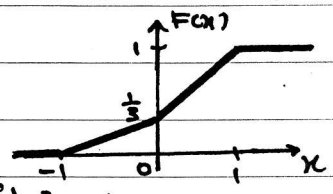
12(a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$: $\frac{1}{3}(1) + \left(\frac{1}{3}+a\right)(1) = 1$

$\therefore\ a = \frac{1}{3}$ ✱

(b) $-1 \le x \le 0$: $P(X \le x) = \int_{-1}^{x} \frac{1}{3}\,dx = \frac{1}{3}(x+1)$

$0 < x \le 1$: $P(X \le x) = \frac{1}{3}(1) + \int_0^x \frac{2}{3}\,dx = \frac{1}{3} + \frac{2}{3}x$

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{3}(x+1) & -1 \le x \le 0 \\ \frac{1}{3}(2x+1) & 0 < x \le 1 \\ 1 & 1 < x \end{cases}$$



(c) $E(X) = \int_{-1}^{0} \frac{1}{3}x\,dx + \int_0^1 \frac{2}{3}x\,dx$

$= \frac{1}{6}\left[x^2\right]_{-1}^{0} + \frac{1}{3}\left[x^2\right]_0^1 = \frac{1}{6}[0-1] + \frac{1}{3}[1-0]$

$= \frac{1}{6}$ ✱

$E(X^2) = \int_{-1}^{0} \frac{1}{3}x^2\,dx + \int_0 \frac{2}{3}x^2\,dx$

$= \frac{1}{9}\left[x^3\right]_{-1}^{0} + \frac{2}{9}\left[x^3\right]_0^1 = \frac{1}{9}[0+1] + \frac{2}{9}[1-0] = \frac{1}{3}$

$\therefore\ s_x = \left(\frac{1}{3} - \left(\frac{1}{6}\right)^2\right)^{1/2} = \frac{\sqrt{11}}{6}$ ✱

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